Do countries like households face limits on what they can borrow? Yes and no: it depends on how you define countries and households. To the extent that countries must “payback” their debt at some point the present value of future trade surpluses must be positive (see Sachs and Larraine Chapter 6 eq. 6.21). In many two period models, all debts are settled in period 2, as if the country had died. Over an infinite horizon this borrowing constraint appears as a transversality condition, an optimization constraint that rules out bubbles and/or Ponzi schemes where new borrowing is used to service existing debt (often there is no investment). But in certain respects countries are not households:

(i) First, countries rarely die (though one can think of infinitely lived households as well, if children willingly assume the debt of their parents) Also a country’s income may grow longer time than a person’s income, especially if it is far from its steady-state output or if one believes in endogenous growth models which imply income can grow forever, there may be a steady state growth rate, but no income steady-state. U.S. income per person for example has grown about 1.6% annually for about 170 years, increasing 15 fold over this period.

(ii) A second difference is that countries are sovereign entities—this limits the recourse of lenders and means countries must be willing as well as able to repay their debt. Hence the limit on what countries can borrow may be much less than what they can actually afford to pay. During the recent housing crisis, some borrowers who could pay defaulted, these are referred to as “strategic defaults” meaning the borrower could pay his or her mortgage but decided to “walk away” typically because the value of the home (collateral) was less than the loan balance. With countries strategic default is always an option and except for assets held abroad, creditors have no recourse (no collateral that can be seized in lieu of payment). And

(iii) Finally, developing countries almost always borrow in a foreign currency (“original sin”) so the real exchange rate becomes a key determinate of their ability to pay.

This handout deals mainly with the first and third aspects of sovereign borrowing. We say external debt is sustainable if it remains a constant fraction of GDP or exports. If debt does not rise as a share of exports or income, it should remain manageable and serviceable. These debt ratios remain constant if debt grows as the same rate as GDP or exports. We use this particular growth rate to derive the trade balance consistent with a stable debt to GDP, \( d_t = D_t/Y_t \) or debt to export ratio \( d_x = D_t/X_t \). Suppose GDP and dollar exports are expected to grow at rates \( g_y \) or exports, \( g_x \), respectively. If debt grows at the same rate as GDP, \( g_y \) or exports, \( g_x \) it as grows along the following path,

\[ D_t = (1 + g)D_{t-1} \tag{1} \]

with \( g = g_y \) or \( g = g_x \) as needed. Since the change in external debt equals the current account deficit,

\[ D_t - D_{t-1} = r^* D_{t-1} - TB_t = -CA_t \quad \text{or} \quad D_t = (1+r) D_{t-1} - TB_t \tag{2} \]

Using (1) we can replace \( D_{t-1} = \) with \( D_t/(1+g_y) \) and then divide by total GDP or \( Y_t \) so that (2) becomes,

\[ \text{tb}^y_t = \left[ \frac{(r^* - g_y)}{(1 + g_y)} \right] \frac{D_t^y}{d_t^y} \tag{3} \]

where \( d_t = D_t/Y_t \) and \( \text{tb}_t = TB_t/Y_t \). If the real interest rate exceeds GDP growth, the rhs of (3) shows the real “burden” of debt service—the share of GDP that must be allocated to debt service to keep the debt ratio from rising. Adding debt service to this special trade balance yields the sustainable current account balance as a share of GDP (assuming we can also predict real \( r^* \) and nominal \( i^* \) world interest rates).
If the real rate of interest is lower than the growth rate, a country can run a trade deficit “forever” as the term in brackets is negative. The above analysis ignores the fact that debt is often denominated in foreign currency (“dollars”). Since exports are invoiced in dollars almost everywhere, a simple way to fix this problem is to focus on the debt to export ratio. To find the trade balance which stabilizes the debt to export ratio, divide (2) by exports and find the trade balance that causes debt to grow at the same rate as exports ($g_x$).

Since exports are reported in current dollars, the nominal world interest rate $i^*$ is most appropriate,

$$\bar{d}_t \left[ \frac{(i^* - g_x)}{(1 + g_x)} \right] = tb_t$$

(4)

To use equations (3) or (4) to find a sustainable current account deficit, choose a target debt to export or GDP ratio, $d^x$ or $d^y$ and then plug the expected growth and interest rates to find the share of GDP or exports that will have to be dedicated to debt service to maintain a constant debt to GDP or export ratio.

The above discussion of debt to GDP ratios ignores secular changes in the real exchange rate. Because LDCs tend to borrow in foreign currencies and because real exchange rates tend to appreciate over time (the Balassa-Samuelson effect) real exchange rate changes should not be ignored. Conversely, during financial crises currency depreciation can increase the debt service burden. Starting with the real exchange rate $q_t = e_t P^*/P_t$, where $e$ is the nominal exchange rate and $P^*$ and $P$ are foreign and domestic prices respectively, equation (1) becomes,

$$q_t d_t = (1 + g_y) q_{t-1} d_{t-1}$$

(1)

where again lower case $d = [eP*/P]D/Y$ that is the debt GDP ratio in local currency and adjusted for changes in domestic and foreign prices, $P$ and $P^*$. As $q_t/q_{t-1} = (1 + \Delta q_t)$ then (1) can also be written as,

$$D_t = \left[ \frac{(1 + g_y)}{(1 + \Delta q_t)} \right] D_{t-1}$$

(5)

showing the evolution of debt in local currency terms $\Delta D_t = D_t - D_{t-1} = -CA_t$, we follow the same procedure as before substituting for $r^* D_{t-1}$ in the current account equation. After a little manipulation we have an expression similar to (3) above, except that it includes the evolution of the real exchange rate.

$$tb_t = \left[ \frac{(\Delta q_t + r^*(1+\Delta q_t) - g_y)}{(1 + g_y)} \right] \bar{d}_t$$

(6)

where again we set our target debt to GDP level $d_t$ and solve for the requisite trade balance % of GDP, $tb_t$, that maintains our desired debt to GDP level, allowing for expected changes in the RER and real GDP. Note that if the real exchange rate is constant ($\Delta q_t = 0$) equation (6) reduces to equation (3) above. A sustained real exchange rate appreciation ($\Delta q_t < 0$) allows a country to run a larger current account deficit and still maintain the same debt to GDP level. Of course a depreciation of the real exchange rate raises the trade surplus necessary to maintain a stable debt to GDP level. $^1$ A “sustainable fiscal deficit” keeps public debt as a constant fraction of GDP, replacing the trade balance is replaced by the primary surplus. The RER ($q$) can even be a factor if the government borrows in dollars. Developing countries can default

or request debt relief (as did highly indebted poor countries with HIPC program) or countries use rapid growth of nominal GDP reduce high debt to GDP ratios over time.

**Sustainable Debt Exercises**

SD-1 Suppose target debt export ratio is 2 and export growth has been averaging 11% annually in nominal terms and its average nominal interest rate on external debt is 8% and the target debt to export rate is 2. Using equation (4) compute the trade balance % of exports that will maintain debt at twice the level of exports. Suppose exports are 25% of GDP, what is the sustainable current account deficit as a % of GDP for this country can run consistent with a constant debt to export ratio? A popular rule of thumb is a 3% of GDP current account deficit is fine, but over 5-6% is dangerous (Mexico’s current account deficit climbed to 8% in 1994). Does the sustainable CA deficit for this country meet this rule of thumb? How would you argue that this deficit is “safe” or would you? Suppose export growth slowed to 5%-- what would happen to the sustainable current account deficit?

SD-2 Suppose a country has a target debt to GDP ratio of 50% and it has been growing at 4% per year and borrowing at a real (world inflation adjusted) interest rate of 3%. Finally, because the productivity of its tradables sector is growing rapidly, its exchange rate is appreciating at 2% per year ($\Delta q_t = .02$). Use eq. (6) to compute the trade balance as a % of GDP that maintains its target debt to GDP ratio (50%)? What happens if its growth prospects change and the real exchange rate $q_t$ is expected to depreciate at 2% per annum? Intuitively, why does an appreciation of the RER increase the sustainable current account deficit as % of GDP?

**SD-3.** What if low growth causes high debt to GDP levels and not vice versa? (one might argue this is what happened to Africa in the 1980s and to the GIPSIs after 2007). How have countries historically escaped very high debt to GDP ratios without formally defaulting (as Argentina did in 2002) see the IMF, WEO, 2013, Chapter 3 The Good, the Bad, and the Ugly: 100 Years of Dealing with Public Debt Overhangs? Hint: This is one area where Rogoff, Krugman (Chapter 10) and the IMF agree.

SD-4 Suppose the debt GDP ratio is one and we want to keep it at one. Discuss some values for GDP and export growth that (a) lead to sustainable CA deficits or be justify our 6% danger point rule of thumb.

\[
tb_t^y = \left[ \frac{(r^* - g_y)}{(1 + g_y)} \right] \bar{d}_t^y
\]

(3)

\[
\bar{d}_t^y \left[ \frac{(i^* - g_x)}{(1 + g_x)} \right] = tb_t^y
\]

(4)

References: