Do countries like households face limits on what they can borrow? Yes and no: when countries stop growing they must “payback” their debt at some point the present value of future trade surpluses must be positive (Sachs and Larraine Chapter 6 eq. 6.21). In two period models, all debts are settled in period 2, as if the country dies. In infinite horizon models this borrowing constraint appears as a transversality condition, a constraint the growth of debt that rules out bubbles and/or Ponzi schemes where new borrowing is used to service existing debt and no real investment. However, countries are not households in the sense that they do invest, grow and restructure, we hope forever. As long as nominal GDP increases, nominal debt can increase too: what economists, investors and the IMF care about is the debt to GDP ratio: as sustainable CA or fiscal deficit is one that keeps D/Y from increasing.

(i) Unlike household heads, countries rarely die (and children can’t sign for their parent’s debt). A country’s income may grow “forever” if it is far from its steady-state potential. If growth is “endogenous” countries can grow forever: U.S. income per person for example has grown about 1.6% annually for 170 years, a 15-fold increase.

(ii) A second important difference is that countries are sovereign entities—this limits the enforcement options of lenders and means countries must be willing to repay their debt. Creditors generally have no recourse (no collateral that can be seized in lieu of payment).

(iii) Any finally, developing countries frequently borrow in a foreign currency (“original sin”) so the real exchange rate becomes a key determinate of their ability to pay exposing themselves and borrowers to currency risk, or currency mismatch. This sort of risks makes currency depreciation (monetary policy) a more problem antidote for coping with sudden stops.

This handout addresses the first and third aspects of sovereign borrowing. External debt is sustainable if it remains a constant fraction of GDP or exports. If debt does not rise as a share of exports or income, it should remain manageable unless there is a large interest rate shock. Debt ratios D/Y remain constant if debt grows as the same rate as GDP or exports. We use a fixed “steady state” growth rate to derive the trade and current account balance consistent with a stable debt to GDP ratio \( \frac{D_t}{Y_t} \) or the debt to export ratio \( \frac{D_t}{X_t} \). We assume GDP and dollar exports grow at rates \( g_y \) or exports, \( g_x \), respectively. If export or GDP or export growth slows, all bets are off. This is how slow growth “causes” a debt crisis. But suppose debt grows at the same rate as GDP, \( g_y \) or exports, \( g_x \) it as grows along the following path,

\[
D_t = (1 + g)D_{t-1} \tag{1}
\]

with \( g = g_y \) or \( g = g_x \) as needed. Since the change in external debt equals the current account deficit,

\[
D_t - D_{t-1} = r^* D_{t-1} - TB_t = -CA_t \quad \text{or} \quad D_t = (1+r) D_{t-1} - TB_t \tag{2}
\]

Using (1) we can replace \( D_{t-1} = \) with \( D_t/(1+g_y) \) and then divide by total GDP or \( Y_t \) so that (2) becomes,

\[
\frac{tb}{(1+g_y)} = \left[ \frac{r^* - g_y}{1+g_y} \right] \frac{d}{D_{t-1}} \tag{3}
\]

where \( d_t = D_t/Y_t \) and \( tb_t = TB_t/Y_t \). If the real interest rate exceeds GDP growth, the rhs of (3) shows the real “burden” of debt service—the share of GDP that must be allocated to debt service to keep the debt ratio from rising. Adding debt service to this special trade balance yields the sustainable current account balance as a share of GDP (assuming we can also predict real \( r^* \) and nominal \( i^* \)world interest rates).

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1/ See the footnote on page 3.
If the real rate of interest is lower than the growth rate, a country can run a trade deficit “forever” as the term in brackets is negative. The above analysis ignores the fact that debt is often denominated in foreign currency (“dollars”). Since exports are invoiced in dollars almost everywhere, a simple way to fix this problem is to focus on the debt to export ratio. To find the trade balance which stabilizes the debt to export ratio, divide (2) by exports and find the trade balance in terms of the debt to export ratio. To use this balance is to find the share of GDP required to maintain a constant debt to GDP ratio. Since exports are reported in current dollars, the nominal world interest rate \(i^*\) is most appropriate.

\[
\overline{d}_t \left[ \frac{(i^* - g_x)}{(1 + g_x)} \right] = \text{tb}_t^* \tag{4}
\]

To use equations (3) or (4) to find a sustainable current account deficit, choose a target debt to export or GDP ratio, \(d^e\) or \(d^y\) and then plug the expected growth and interest rates to find the share of GDP or exports that will have to be dedicated to debt service to maintain a constant debt to GDP or export ratio.

The above discussion of debt to GDP ratios ignores secular changes in the real exchange rate. Because LDCs tend to borrow in foreign currencies and because real exchange rates tend to appreciate over time (the Balassa-Samuelson effect) real exchange rate changes should not be ignored. Conversely, during financial crises currency depreciation can increase the debt service burden. Starting with the real exchange rate \(q_t = e_t P^*/P_t\), where \(e\) is the nominal exchange rate and \(P^*\) and \(P\) are foreign and domestic prices respectively, equation (1) becomes,

\[
q_t d_t = (1 + g_y) q_{t-1} d_{t-1} \tag{5}
\]

where again lower case \(d = [(q^D)/Y]\) that is the debt GDP ratio in local currency and adjusted for changes in domestic and foreign prices \(P\) and \(P^*\), where \(q = eP^*/P\). Since \(q_t/q_{t-1} = (1 + \Delta q_t)\) then (1) can also be written as,

\[
D_t = \left[ \frac{(1 + g_y)}{(1 + \Delta q_t)} \right] D_{t-1} \tag{6}
\]

showing the evolution of debt in local currency terms \(\Delta D_t = D_t - D_{t-1} = -CA_t\), we follow the same procedure as before substituting for \(r^*D_{t-1}\) in the current account equation. After a little manipulation we have an expression similar to (3) above, except that it includes the evolution of the real exchange rate.

\[
tb_t = \left[ \frac{(\Delta q_t + r^*(1+\Delta q_t) - g_y)}{(1 + g_y)} \right] \overline{d}_t \tag{7}
\]

where again we set our target debt to GDP level \(d_t\) and solve for the requisite trade balance % of GDP, \(tb_t\), that maintains our desired debt to GDP level, allowing for expected changes in the RER and real GDP. Note that if the real exchange rate is constant (\(\Delta q_t = 0\)) equation (6) reduces to equation (3) above. A sustained real exchange rate appreciation (\(\Delta q_t < 0\)) allows a country to run a larger current account deficit and still maintain the same debt to GDP level. Of course a depreciation of the real exchange rate raises the trade surplus necessary to maintain a stable debt to GDP level. A “sustainable fiscal deficit” keeps public debt as a constant fraction of GDP, replacing the trade balance is replaced by the primary surplus.

\[\text{For derivation of a very similar condition see Frenkel and Razin (1996) \textit{Fiscal Policy and Growth in the World Economy chapter 17 page 517 equation 17.6}. \textit{Gillis et. al 5\textsuperscript{th} ed. (1996) derive an expression similar to (4) on page 414, equation 15-2.}]}\]
The RER (q) can even be a factor if the government borrows in dollars. Developing countries can default or request debt relief (as did highly indebted poor countries with HIPC program) or countries use rapid growth of nominal GDP reduce high debt to GDP ratios over time.

**Sustainable Debt Questions**

**SD-1** Suppose target debt export ratio is 2 and export growth has been averaging 11% annually in nominal terms and its average nominal interest rate on external debt is 8% and the target debt to export rate is 2. Using equation (4) compute the trade balance % of exports that will maintain debt at twice the level of exports. Suppose exports are 25% of GDP, what is the sustainable current account deficit as a % of GDP for this country can run consistent with a constant debt to export ratio? A popular rule of thumb is a 3% of GDP current account deficit is fine, but over 5-6% is dangerous (Mexico’s current account deficit climbed to 8% in 1994). Does the sustainable CA deficit for this country meet this rule of thumb? How would you argue that this deficit is “safe” or would you? Suppose export growth slowed to 5%— what would happen to the sustainable current account deficit?

**SD-2** Suppose a country has a target debt to GDP ratio of 50% and it has been growing at 4% per year and borrowing at a real (world inflation adjusted) interest rate of 3%. Finally, because the productivity of its tradables sector is growing rapidly, its exchange rate is appreciating at 2% per year (Δq = -.02). Use eq. (6) to compute the trade balance as a % of GDP that maintains its target debt to GDP ratio (50%)? What happens if its growth prospects change and the real exchange rate q_t is expected to depreciate at 2% per annum? Intuitively, why does an appreciation of the RER increase the sustainable current account deficit as % of GDP?

**SD-3.** What if low growth causes high debt to GDP levels and not vice versa? (one might argue this is what happened to Africa in the 1980s and to the GIPSIs after 2007). How have countries historically escaped very high debt to GDP ratios without formally defaulting (as Argentina did in 2002) see the IMF, WEO, 2013, *Chapter 3 The Good, the Bad, and the Ugly: 100 Years of Dealing with Public Debt Overhangs?* Hint: The U.S. and Japan and most countries inflate their way out of debt. This is where *Rogoff*, Krugman, 2009 (Chapter 10) and the IMF agree.

**SD-4** Suppose the debt GDP ratio is one and we want to keep it at one. Discuss some values for GDP and export growth that (a) lead to sustainable CA deficits or be justify our 6% danger point rule of thumb.

\[
\begin{align*}
\text{tb}_t^y &= \left[\frac{(r^* - g_y)}{(1 + g_y)}\right] d_t^y \\
\text{d}_t^* &= \left[\frac{(i^* - g_x)}{(1 + g_x)}\right] = \text{tb}_t^y
\end{align*}
\]

Footnote from first page: 1/ Hence the limit on what countries can borrow may be much below what they can afford to pay. Sovereign credit ratings reflect both countries ability and willingness to may (Hence center right governments may be able to borrow more). During housing crises, some U.S. borrowers who could pay defaulted, “strategically” deciding to “walk away” when the value of the home (collateral) was less than the loan balance (but not in Spain). With countries strategic default is always an option and except for assets held abroad (e.g. Venezuela’s PDVSA owns CITGO in the U.S.) An exception is the Libertad, an Argentine Navy sailing ship seized in Accra by NML Capital in 2012.
References:

Abbas, SM Ali, Mr B Akitoby, Mr J R. Andritzky, Mr H Berger, Mr T Komatsuzaki, and J Tyson (2013) *Dealing with high debt in an era of low growth*, No. 13-17. International Monetary Fund, “Its mostly fiscal” slides type the title into GS to get many pdf versions…


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https://www.youtube.com/results?search_query=public+debt+sustainability+1

https://www.youtube.com/watch?v=xPjJ5fOoCuM
Debt Sustainability Analysis

What are the tools to assess debt sustainability? How can countries effectively manage their sovereign debt? To answer these questions, this course combines theory with hands-on exercises.

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https://www.edx.org/course/debt-sustainability-analysis-imfx-dsax-4
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Inflation is Now the Lesser Evil

Dec 2, 2008 KENNETH ROGOFF  It is time for the world’s major central banks to acknowledge that a short burst of moderate inflation would be extremely helpful in unwinding today’s epic debt morass. It may not be fair, but fear of inflation, when viewed in the context of a possible global depression, is like worrying about getting the measles when one is in danger of getting the plague.

https://www.project-syndicate.org/commentary/inflation-is-now-the-lesser-evil?barrier=accessreg