ECGA 6470 **Growth and Development** **Spring 2014**

An AK model with government spending

Barro (1990) adds a surprisingly benevolent government to an endogenous "AK" growth model. Rather than government consumption financed by distortionary taxes as in Easterly (2005) government spending on public investment in roads, ports, sanitation, schools, etc. is financed by uniform income tax. The public sector budget is always balanced and public investment complements private investment so higher taxes and more government spending may be associated with an increase or a decrease in growth. In fact the public sector must tax and invest or growth will be very low. The only problem is that the private sector ignores the additional tax revenues and public investment generated by its private investment, so it may invest too little. Because this is basically an AK model, it provides a very tractable framework modeling imported investment goods for example, as in Basu and McLeod (1992) or imported workers for example. Barro (1990) adds public investment or capital g to the AK model where y is GDP per person, $y = Ak^{1-\alpha}g^{\alpha}$ and spending equals tax revenues $g = \tau y$, where g is government spending per capita is determined by the tax rate τ since the budget is always balanced. Using the government budget constraint we replace g in (1A) with τy to obtain, $y = A(\tau y)^{\alpha}k^{1-\alpha} = A\tau^{\alpha}y^{\alpha}k^{1-\alpha}$. Bringing y^{α} over to the LHS we have $y^{1-\alpha} = A\tau y^{\alpha}k^{1-\alpha}$. Raising both sides by 1/(1-α) yields

$y = \left(A^{\frac{1}{1-\alpha}} \tau^{\frac{\alpha}{1-\alpha}} \right) k = A^*(\tau)k$ where the new MPK in brackets $A^*(\tau)$ plays a role analogous the A in the AK model, except that A* depends on the tax rate τ. One might think that as before the growth rate $\gamma = 1/\theta\,[A^* - \rho]$ where the intertemporal elasticity of substitution is 1/θ and ρ is the discount rate. But recall that the private sector only receives after tax income on capital investment, or $(1-\tau)(1-\alpha)A^*(\tau)$. Note that government has two effects on private income: it reduces after tax private income (1-τ) and up to a point greater g raises the marginal product of capital $A^*(\tau)$ for any given private capital stock k. At some point these two effects balance out and we obtain the growth maximizing tax rate τ which turns out to be α, the optimal share of g implied by our Cobb-Douglas production function, see Figure 1 below. This is analogous to the golden rule savings rate s = α. The growth maximization problem has another twist however. Since the private sector does not take into account that the taxes it pays income generated by its investment increases g, the private sector will under invest and growth will be too low. The socially optimal growth rate is $(1-\alpha)A^*(\tau) > (1-\tau)(1-\alpha)A^*(\tau)$ so the decentralized private sector determined growth rate is lower that the "central planners" optimal growth rate. Note also that if τ = 0, g = 0 and we have a public investment poverty trap as the private MPK = 0 and the growth rate is negative. Of course if the government takes the whole national product (τ = 1) the growth rate also goes to zero since there is no private investment. The optimal tax rate is obviously somewhere in between 0 and 1, as shown in Figure 1 below. We cannot say *a priori* whether a small government or a large government is good for economic growth, it depends on given technology parameter α.

To solve the households (and planners) optimal growth problem more explicitly, assume infinitely lived households own firms and produce output using a production function that benefits from per capita infrastructure investment g (roads, ports, dams, etc). Households maximize a constant relative risk aversion or CRRA utility $u(c_t)$,

$$\max \int_0^\infty \frac{c_t^{1-\theta}}{1-\theta} e^{-\rho t} dt \quad \textit{subject to} \quad \dot{k}_t = (1-\tau) y_t - c_t \text{ where } y_t = A k_t^{1-\alpha} g_t^\alpha \text{ and } 0 < \alpha < 1.$$

Per capita public spending on infrastructure g is financed by a proportional tax on income so $g = \tau y$. The government does not borrow so the budget is always balanced. All variables are per capita but note that labor is not required to produce y but we assume k includes human as well as physical capital. To solve maximization problem of a representative, infinitely lived private agent who controls k and consumes c taking g and τ as given (set by the government) we can use a present value Hamiltonian solving for the present value of consumption subject to the accumulation constraint in brackets:

$$H = \frac{c_t^{1-\theta}}{1-\theta} e^{-\rho t} + \mu\left[(1-\tau) y_t - c_t\right] \textit{ where } y_t = A k_t^{1-\alpha} g^\alpha .$$

Since the private sector only controls k, we differentiate H with respect k_t and c_t only to obtain the first order conditions (FOCs):

$$H_c: \quad c_t^{-\theta} e^{-\rho t} - \mu = 0 \qquad (1)$$
$$H_k: \quad \mu\left[(1-\tau)(1-\alpha) A k^{-\alpha} g^\alpha\right] = \dot{\mu} \qquad (2)$$
$$H_\mu: \quad \left[(1-\tau) y_t - c_t\right] = \dot{k} \quad \textit{where again, } y_t = A k_t^{1-\alpha} g^\alpha$$

The transversality condition is $\lim_{t\to\infty}\left[\mu(t)\cdot k(t)\right] = 0$ which the dynamic version of the Inada conditions. To solve for the growth rate of consumption first take log of (1) to obtain:

$-\theta * \ln c_t - \rho * t = \ln \mu .$ Taking the time derivative of this equation yields:

$$-\theta \cdot \frac{\dot{c}}{c} - \rho = \frac{\dot{\mu}}{\mu} \qquad (3)$$

Dividing both side of (2) by μ gives $\left[(1-\tau)(1-\alpha) A k^{-\alpha} g^\alpha\right] = \frac{\dot{\mu}}{\mu}$. Using this expression to replace the RHS of (3) yields: $\left[(1-\tau)(1-\alpha) A k^{-\alpha} g^\alpha\right] = -\theta \cdot \frac{\dot{c}}{c} - \rho$

Solving for $\frac{\dot{c}}{c}$ we get the decentralized competitive solution:

$$\frac{\dot{c}}{c} = \frac{1}{\theta}\left(\underbrace{\left[(1-\tau)(1-\alpha) A k^{-\alpha} g^\alpha\right]}_{MPK} - \rho \right) \quad \textit{or} \quad g = \frac{\dot{c}}{c} = \frac{1}{\theta}\left(A^* - \rho\right) \textit{ where } A^* = (1-\tau)(1-\alpha) A k^{-\alpha} g^\alpha \qquad \textbf{(4)}$$

This expression says the growth in consumption depends on (i) the gap between the marginal product of capital and the rate of time preference, ρ and (ii) on the IES 1/θ. The higher the θ, the more value individuals place on smoothing consumption. The higher θ is, the higher return to capital (MPK) it takes to encourage investment which is what raises the growth rate of consumption (and output y).

It appears that the blue MPK term in (4) depends on k, but it does and should not since this an AK class endogenous growth. To see this recall that $g = \tau Y$ so that $g = \tau \cdot Ak^{1-\alpha} g^{\alpha}$, solving for g yields,

$g = \left(\tau \cdot Ak^{1-\alpha}\right)^{\frac{1}{1-\alpha}} = (\tau \cdot A)^{\frac{1}{1-\alpha}} k$ which implies that $g^{\alpha} = (\tau \cdot A)^{\frac{\alpha}{1-\alpha}} k^{\alpha}$. Substituting g^{α} into the blue MPK term above causes the k terms to drop out leaving, **MPK =** $(1-\tau)(1-\alpha)A^{\frac{1}{1-\alpha}}\tau^{\frac{\alpha}{1-\alpha}}$ **= A*(τ)**

Hence the after-tax marginal product of capital calculated above, **A*(τ)**, plays the same role as the A in the AK model (except that it depends on the tax rate). The expression consumption growth equation (4) shows that government affects the marginal product of capital through two channels: increases in g raise the MPK up to a point, but taxes always reduce the private return to capital, the task of a good government is balance these two effects:

$$MPK = \underbrace{(1-\tau)}_{negative\ effect\ of\ taxation} \quad (1-\alpha)Ak^{-\alpha} \quad \underbrace{g^{\alpha}}_{positive\ effect\ of\ public\ services} . \qquad (5)$$

To solve the model from the point of view of a benevolent growth maximizing government (central planner) we know that the planner satisfies the condition δy/δg =1 ("marginal product of government spending" or MPG). The government should provide public investment or services until the MPG equals one. So the MPG must equal government spending *g:*

$$\underbrace{\alpha Ak^{1-\alpha} g^{\alpha-1}}_{MPG} = \underbrace{g}_{spending}$$

Solving for g we find that $1/\alpha = (y/g) = 1/\tau$, so that **α = τ** when MPG =1 .

The key difference between the decentralized and the centralized solution, is that the firm responds to the after-tax private marginal product MPK= (1-τ)∂y/∂k while the planner takes into account the social marginal product (∂y/∂k): additional private income also raises taxes and g as well as k. To get the social planner's growth rate, eliminate the (1-τ) from (5) to find the planner's MPK and we get that MPK = $(1-\alpha)A^{\frac{1}{1-\alpha}}\tau^{\frac{\alpha}{1-\alpha}}$ where substituting **α = τ** yields MPK = $(1-\alpha)A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}$

So the planner should set the tax rate to maximize growth rate and the red MPK,

$$\frac{\dot{c}}{c} = \frac{1}{\theta}\left(\underbrace{\left[((1-\alpha)A^{\frac{1}{1-\alpha}}\tau^{\frac{\alpha}{1-\alpha}} \right]}_{MPK} - \rho \right). \qquad (6)$$

Note the MPK in the decentralized solution is (1-τ)(∂y/∂k), it is smaller than the social marginal product ∂y/∂k, because of the tax rate. This gap between social and private returns leads to a lower growth rate in the decentralized solution summarized by (4) compared to the socially optimal growth shown in eq. (6). Note however, that whether the relevant growth rate is dictated by (4) or (6), the growth maximizing tax rate is **τ** = α**.** For the decentralized solution, a benevolent government would want to maximize the utility of individuals with a certain size of the government, *g*. To find this, we differentiate the following growth rate (combination of 4 and 5) with respect to **τ:** $\frac{\dot{c}}{c} = \frac{1}{\theta}\left(\underbrace{\left[(1-\tau)(1-\alpha)A^{\frac{1}{1-\alpha}}\tau^{\frac{\alpha}{1-\alpha}} \right]}_{MPK} - \rho \right)$

Differentiating: $\partial(\dot{c}/c)/\partial\tau = \left[-1\left((1-\alpha)A^{\frac{1}{1-\alpha}}\tau^{\frac{\alpha}{1-\alpha}} \right) \right] + (1-\tau)(1-\alpha)\left(\frac{\alpha}{1-\alpha} \right) A^{\frac{1}{1-\alpha}}\tau^{\frac{-1}{1-\alpha}} = 0$

Simplifying: $\left[\left((1-\alpha)A^{\frac{1}{1-\alpha}}\tau^{\frac{\alpha}{1-\alpha}} \right) \right] = (1-\tau)\left(\frac{\alpha}{1-\alpha} \right)(1-\alpha)A^{\frac{1}{1-\alpha}}\tau^{\frac{-1}{1-\alpha}}$,

$(1-\alpha)\tau = \alpha(1-\tau)$, which can only hold if $\tau = \alpha$. The condition that **τ =α** corresponds to the golden rule savings rate in the Solow model (recall that s = α in problem set #1) here the tax rate is the savings rate for public owned capital, so α is the optimal tax rate in both solutions. Graphically, the optimal tax rate **τ** = α is

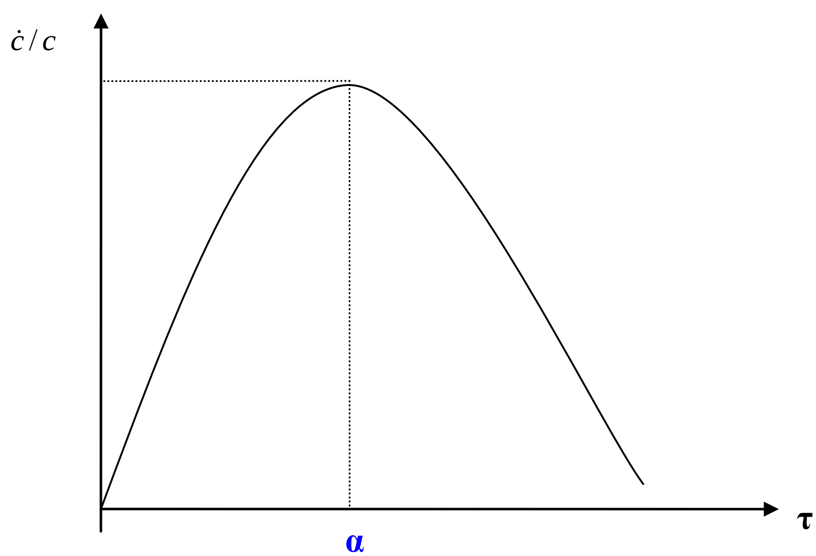
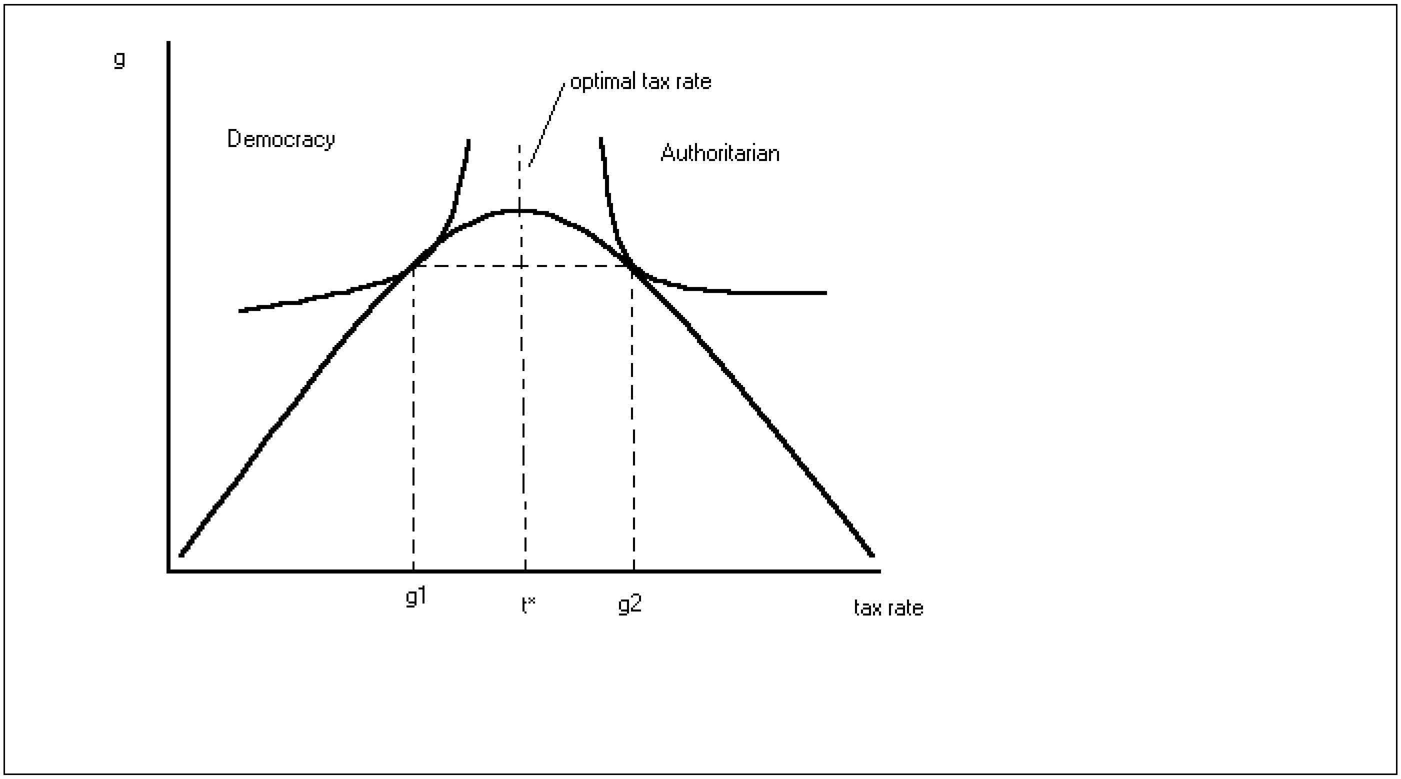


Figure 1: The Growth maximizing tax rate is α

A note on Democracy and Economic Growth

There is a large and inconclusive literature on whether democracies have higher growth rates (see Barro, 1999 and Rodrik, 1999). Assuming taxpayers vote and paying taxes is painful (distortionary) the optimal tax rate for a democracy is likely to be a bit lower than with no frictions (lower than α in the above example). If voters (taxpayers) have no influence on the other hand, and authoritarian rulers benefit directly from higher tax revenues (e.g. higher rents or salaries) the optimal tax rate for an authoritarian regime is likely to be higher than α. The implication of these arguments is summarized in the diagram below, the democratic and authoritarian regime and may have similar growth rates, but the government sector is likely to be smaller with a democratic regime. This would be an interesting hypothesis to test, all else equal.



Further Reading:
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