2. *Can differences in the utilization of factors of production explain differences in TFP?* Consider a production function of the form $Y = IK^{\alpha}(hL)^{1-\alpha}$, where I denotes total factor productivity and the other notation is standard. Suppose I varies by a factor of ten across countries, and assume $\alpha = 1/3$.

   (a) Suppose differences in infrastructure across countries lead only to differences in the fraction of physical capital that is utilized in production (vs. its use, say, as fences to protect against diversion). How much variation in the utilization of capital do we need in order to explain the variation in TFP?

   (b) Suppose both physical capital and skills vary because of utilization, and for simplicity suppose that they vary by the same factor. How much variation do we need now?

   (c) What do these calculations suggest about the ability of utilization by itself to explain differences in TFP? What else could be going on?

3. *Social infrastructure and the investment rate.* Suppose that rates of return to capital are equalized across countries because the world is an open economy, and suppose that all countries are on their balanced growth paths. Assume the production function looks like $Y = IK^{\alpha}L^{1-\alpha}$, where I reflects differences in social infrastructure.

   (a) Show that differences in I across countries do not lead to differences in investment rates.

   (b) How might social infrastructure in general still explain differences in investment rates?

4. Discuss the meaning of the quotation that began this chapter.

8 POPULATION AND THE ORIGIN OF SUSTAINED ECONOMIC GROWTH

> This [law of our nature] implies a strong and constantly operating check on population from the difficulty of subsistence. This difficulty must fall some where and must necessarily be severely felt by a large portion of mankind. . . . And the race of man cannot by any efforts of reason, escape from it . . . misery is an absolutely necessary consequence of it.
>
> —THOMAS MALTHUS, 1798

> The denser the population the more minute becomes the subdivision of labor, the greater the economies of production and distribution, and hence the very reverse of the Malthusian doctrine is true . . .
>
> —HENRY GEORGE, 1879

We have assembled a model that explains the long-run growth rate of technology and income per capita. Surprisingly, both are driven by the population growth rate. The number of ideas that an economy can generate is related to the number of people, and ultimately living standards improve with the size of the population.

This is surprising because the logic of Thomas Malthus, captured in the quotation above, appears so compelling. Increases in population size should, given a fixed supply of natural resources, drive down living standards. Malthus, though, overlooked what Henry George appreciated—the capacity of people for innovation.

The failure of Malthus to acknowledge the potential benefits of a larger population is ironic, given that he was living in England at the very cusp of the Industrial Revolution that would demolish his predictions. In Malthus's defense, in 1798 real wages in England had not grown for two hundred years, were lower than they had been in 1500, and were equivalent to the real wages in the year 1200 (Clark 2005). Henry George, in contrast, had the benefit of looking back at one hundred years of growth in living standards across Western Europe, growth that occurred despite historically fast population growth across the continent.

In this chapter we will build endogenous population growth into our model of economic growth, linking decisions regarding the number of children to have to the income level. In addition, we'll explicitly incorporate a fixed stock of resources—land—into the production function. This will allow for "Malthusian" dynamics, where increasing population size drives down living standards.

Combining this with our earlier work linking population size to the rate of innovation, we'll be able to provide an explanation for the transition from a low-income, low-growth world prior to roughly 1800 to the high-income, high-growth world that we live in today. The dynamics of the population growth rate will be key to this explanation, and we'll describe the microeconomics behind choices in family size that drive this growth.

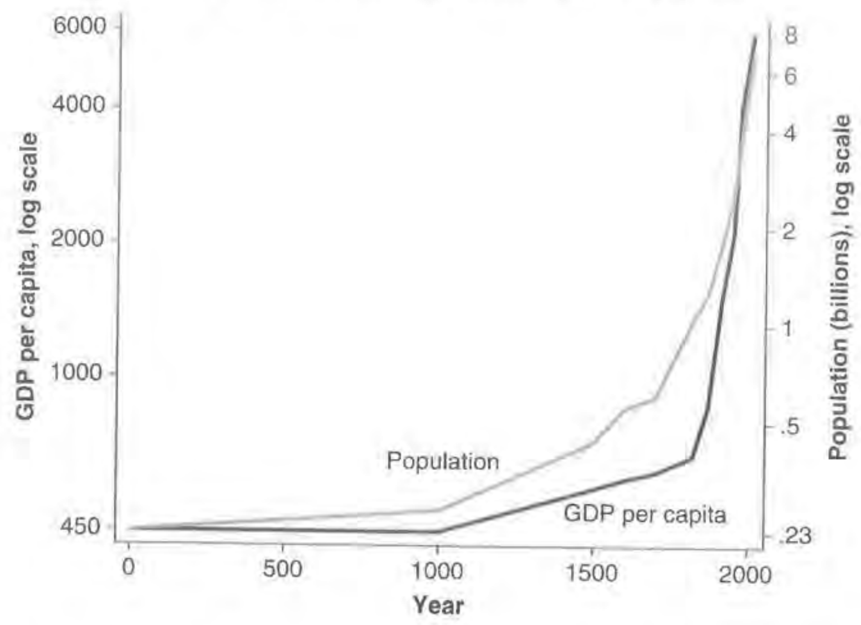
8.1 POPULATION AND LIVING STANDARDS

We can break up the history of human population growth and education into three eras, following the work of Galor and Weil (2000).

Figure 8.1 plots both income per capita and total population for the world from the year 0 through 2010. The figure, in some sense, does some injustice to the history of both population and GDP per capita, as humans have been a distinct species since roughly 1 million BCE. A full graph would extend backward to that point in time, and the rapid expansion of both population and income per capita after around 1750 would become a blip on the graph at the very end.

To put into perspective how little Figure 8.1 actually captures, suppose we were to map out world history on a football field. Let the goal

WORLD INCOME PER CAPITA AND POPULATION



SOURCE: Maddison (2010).

line on one end of the field stand for 1 million BCE. Let the other goal line correspond to 2000 CE. Humans were essentially hunters and gatherers for the overwhelming majority of history, until the development of agriculture approximately ten thousand years ago. On our football field, hunting and gathering occupies that first 99 yards of the 100-yards field; systematic agriculture only begins on the one-yard line. The year 1 CE is only 7 inches from the goal line, and the Industrial Revolution begins less than one inch from the goal line. In the history of humankind, the era of modern economic growth is the width of a golf ball perched at the end of a football field.

If we were to extend Figure 8.1 back to 1 million BCE, two trends would show up. First, the size of the population would continue to shrink as we went farther back in time. In 1 million BCE, estimates put the total number of human beings at only 125,000. This increases to about 230 million by the start of our figure in the year 0 CE. This is a growth rate of only 0.0007 percent per year.[1] The second trend would be the stagnation in income per capita. On the figure, this is measured as equivalent to roughly 450 dollars per year (at today's values) in the year zero. This does not fall as we move backward in time. Building off of evidence from surviving foraging tribes, Gregory Clark (2007) estimates that prehistoric hunters and gatherers consumed just as much food per day as individuals alive around 0 CE. Furthermore, food consumption did not change demonstrably from this level until around 1800.

In the following sections we'll provide more detail on the growth in both population and income per capita seen in Figure 8.1. The limitation to years after 0 CE is due to a lack of regular data prior of this point in time, but one should keep in mind that we can extend the description back for thousands and thousands of years.

8.1.1 THE MALTHUSIAN ERA

The period from the origin of modern humans in 1 million BCE to 1800 CE is referred to as the Malthusian era, after the author of the opening quotation to this chapter. By the reckoning of Angus Maddison (2008), average income per capita was around $450 per year across the entire globe in 1 CE, and did not grow at all between 1 CE and 1000 CE. From 1000 to 1820 CE, average income per capita grew to $670 per year, a growth rate of only 0.05 percent per year. By 1820 a divergence across countries was already evident—the richest Western European nations had an income per capita of around $1,200, but even this implies only a growth rate from 100 to 1820 of 0.14 percent per year.

At the same time that income per capita was low and barely growing, the population of the world was also low and barely growing. Between the year 0 and 1000 CE, total world population went from 230 million to 261 million, a growth rate of just 0.02 percent per year. Note that, while low, this is *twenty-nine* times the growth rate from 1 million BCE to 0 CE. After 1000, population grew at 0.1 percent per year until it was 438 million in

[1] This example illustrates the remarkable power of compounding; even at this near-zero growth rate, world population increased more than a thousandfold over this million-year period.

1500 CE, and then by 0.27 percent per year until total population was 1.04 billion in 1820.

During the Malthusian Era, there was also little effort spent accumulating formal human capital. While universities were founded in Europe as early as the eleventh and twelfth centuries, these were limited to a very small class of individuals. Education in this era, to the extent that is was provided, seemed to serve mainly cultural and political purposes (Landes 1969).

8.1.2 THE POST-MALTHUSIAN ERA

Around 1800 CE, there is a notable acceleration in both income per capita and population growth rates. This began first in Europe and its offshoots in North America, and then later across different areas of the world. Where the Malthusian era was characterized by very low population growth rates, the post-Malthusian era saw a surge in population growth. Between 1820 and 1870, population growth averaged 0.4 percent a year, followed by a growth rate of 0.8 percent per year from 1870 to 1913 and 0.9 percent from 1919 to 1950. These are already roughly four times higher than in the Malthusian era. With most countries in the world still passing through the post-Malthusian era, the growth rate of world population increased to 1.9 percent from 1950 to 1973.

At the same time that more children were being born and surviving to adulthood, these children were not necessarily receiving any formal education. Even after the arrival of the Industrial Revolution in England, by 1841 only 5 percent of male workers and 2 percent of female workers worked in industries in which literacy was required (Mitch 1992).

The key feature of this era that differs from the Malthusian era, though, is that the acceleration of population growth rates did not lead to declining living standards. In contrast, this is the period in which growth in income per capita begins to rise appreciably, as can be seen clearly in Figure 8.1. Growth in world income per capita rose to 0.5 percent per year from 1820 to 1870 and 1.3 percent per year from 1870 to 1913. These are rates ten times higher than those in the Malthusian era. In the leading areas of Western Europe and its offshoots, growth ran ahead of even these rates.

8.1.3 THE MODERN GROWTH ERA

The final era captures the developed world today, as well as those countries that are quickly converging toward those living standards. From a population perspective, there are two main features of the modern growth era.

The first, and most dramatic, perhaps, is the demographic transition. After the surge in population growth in the post-Malthusian era, countries began to see fertility behavior shift toward smaller families. Beginning in Western Europe and North America, population growth began to fall in the early 1900s, declining by over half between 1870 and 1950. This was due, in large part, to declines in the total fertility rate (TFR), a measure of the average number of children born per woman. Around 1870, the TFR was as high as 6 in the Netherlands and Germany, 5.5 in England, and 4 in France. By the 1970s, the TFR was right around 2 across Western Europe, implying that the population growth rate was becoming close to 0.[2]

Different regions of the world have entered their own demographic transitions, differing only in the timing. In Latin America the transitions began in the middle of the twentieth century, with Asia close behind. Africa currently has population growth rates that have stopped rising, perhaps indicating that this continent is about to enter a demographic transition of its own.

At the same time that population growth rates started to decline from their peaks, those children who were being born were starting to acquire higher levels of education. Leaders, such as the United States and the Netherlands, had achieved universal primary school education by the middle of the nineteenth century, while in the rest of Western Europe this did not occur until nearly 1900. Widespread secondary schooling first spread through the United States in the early twentieth century, but even by the 1960s the average education in Western Europe was only about six years. While education levels differ widely across countries, across the second half of the twentieth century most areas have seen significant growth in the average years of schooling.

[2] A second key component of the demographic transition is the steep decline in mortality rates that often precedes the decline in fertility. We do not dwell specifically on mortality processes, but doing so would not alter the general model of population processes that we develop below.

As can be seen in Figure 8.1, growth in income per capita continues during this era. As the modern growth era began, growth was very fast, so that income per capita is rising more quickly than at any time in history. Following that, there has been some tendency for the growth rate to decline slightly as the world enters the twenty-first century.

8.2 THE MALTHUSIAN ECONOMY

How do we explain the differences in the growth of living standards and population in the different eras, and what is it that drives the transition from one to the next? The model we developed in Chapters 2 through 5 took population growth as exogenous, and implied that income per capita would only be stagnant if population growth was zero. However, the evidence of the Malthusian era is that living standards did not grow, yet the population size was rising continually.

8.2.1 PRODUCTION WITH A FIXED FACTOR

To describe the economics at work in this era, we will introduce a production function that replaces the physical capital stock with land. In the Malthusian era the vast majority of production was agricultural, and land represented the most important factor of production along with labor.[3] The important element of Malthusian economy is that land is in fixed supply.

Let the production function be

$$Y = BX^{\beta}L^{1-\beta}, \tag{8.1}$$

where X is the stock of land, and L is the size of population. B denotes the level of technology; B multiplies the entire production function rather than just augmenting labor as this will simplify the analysis. The production function exhibits constant returns to scale in the rivalrous

[3] We could include both capital and land as factors of production, but this would complicate the explanation without changing the ultimate results.

inputs of land and labor, reflecting the standard replication argument. If we were to create a replica of the economy, with an identical amount of land and people, then overall output would double.

Dividing both sides by L gives us income per capita,

$$y = B\left(\frac{X}{L}\right)^{\beta}, \tag{8.2}$$

which has the key property that y is inversely related to the size of the population. If L rises, then aggregate output will rise, but because of decreasing returns, output per capita will fall. Essentially, more people are trying to work with the fixed supply of land, X, and they are becoming more and more crowded, reducing everyone's output.

Up to now, we have presumed that population, L, grows at an exogenous rate. In the Malthusian model, if L continues to grow forever, then output per capita will eventually be driven to zero. While for much of history the average person was very poor, it is not the case that he or she consumed literally *nothing*. To accommodate this, the Malthusian model makes population growth endogenous.

Specifically, the Malthusian model assumes that population growth is increasing with income per capita. It is easiest to conceive of this relationship by thinking of food as the main output of the economy, consistent with the evidence of the Malthusian era. At very low levels of income, food is scarce and nutrition is poor. Families have difficulty conceiving and infant mortality is high. As income rises, families have more food available, and the capacity to have children and keep them alive through childhood improves.

Mathematically, we can represent this Malthusian population process as

$$\frac{\dot{L}}{L} = \theta(y - \underline{c}), \tag{8.3}$$

where $\underline{c}$ represents a "subsistence level" of consumption and θ is a parameter governing the response of population growth to income. Note that it is quite possible for $\dot{L}/L$ to be negative. If y is sufficiently small, then people have incomes below the subsistence level and their families do not have enough surviving children to replace the parents, and the population declines.

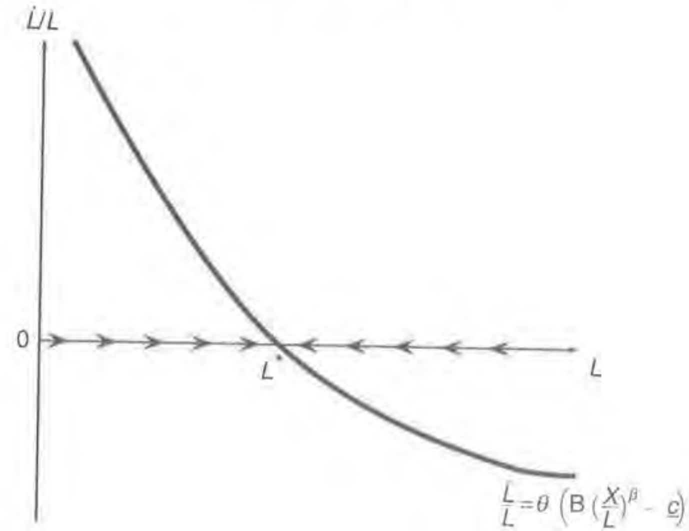
Combining this population process with the expression for income per capita above yields

$$\frac{\dot{L}}{L} = \theta\left(B\left(\frac{X}{L}\right)^{\beta} - \underline{c}\right). \tag{8.4}$$

That is, the growth rate of population is negatively related to the size of population itself. Figure 8.2 plots this function, showing clearly that for low levels of population, people are relatively rich, and population growth is positive. For large levels of L, however, income per capita is very low, and the population growth rate is actually negative.

What can also be seen in Figure 8.2 is that for a specific population size, L^*, population growth is exactly zero. If population is equal to L^*, then the population neither grows nor shrinks, and stays at exactly L^*. This is referred to as the Malthusian steady state, the population size that can be sustained indefinitely.

FIGURE 8.2 MALTHUSIAN DYNAMICS OF POPULATION



Note: The figure shows the negative relationship between population growth $\dot{L}/L$ and population size L. Because of this negative relationship, the population size will tend to grow if less than L^*, and will shrink if more than L^*, so that in the long run population will be equal to L^*.

Importantly, the dynamics ensure that the economy always ends up at L^* no matter where it starts. If population size is less than L^*, then what can be seen in the figure is that $\dot{L}/L > 0$. Population size grows, and so long as L remains smaller than L^*, it will continue to grow. On the other hand, if population size is greater then L^*, then we see that $\dot{L}/L < 0$, and population size is shrinking. It will continue to shrink so long as L is greater than L^*.

We can use equation (8.4) to solve for the actual size of L^*. Setting $\dot{L}/L = 0$ in that equation, this results in

$$L^* = \left(\frac{B}{\underline{c}}\right)^{1/\beta} X. \tag{8.5}$$

The steady-state population is proportional to X, the amount of land. Larger land areas would be capable of supporting larger populations, somewhat unsurprisingly. In addition, if technology (B) increases, this increases the size of the steady-state population as well. Higher technology means that the economy can support more people on the same area of land because it makes that land more productive. The greater the subsistence requirement, the smaller is the steady-state population.

While population size is dictated by the resources available and the technology level, examining equation (8.3) shows that living standards are not. Setting $\dot{L}/L = 0$ in that equation, we can solve for

$$y^* = \underline{c}. \tag{8.6}$$

That is, income per capita in steady-state is dictated solely by the subsistence level of consumption. It does not respond to either X or B.

What is happening here that neither resources nor technology have any impact on living standards? This is a result of having population growth positively related to income per capita. If y were greater than y^*, then people would have relatively large families, $\dot{L}/L$ would be greater than zero, and population size would increase. However, given fixed levels of X and B, increasing the number of people lowers output per capita, y. So any time the economy does have relatively high living standards, fertility rates rise and the economy literally eats away at its own prosperity. This reaction is precisely what Thomas Malthus was describing in 1798, and why he predicted that living standards were doomed to remain stagnant in the long run.

One of the other implications of the Malthusian model is that any exogenous decline in population will temporarily raise living standards. For example, when the Black Death tore through Europe in the fourteenth century, it killed somewhere between 30 and 50 percent of the population. This major decline in the number of people meant that those remaining had access to a greater stock of resources per capita. As a consequence, living standards increased dramatically. Clark (2007) reports real wages in England doubling between 1350 and 1450, while in Italy wages grew two-and-a-half times larger in the same period. These increased living standards, however, did not last. By the 1500s real wages across Europe were back to pre-Black Death levels.

The return of living standards to their pre-Black Death levels was coincident with the recovery of population to its previous size, consistent with the Malthusian model. Italy's population was 10 million in the year 1300, prior to the Black Death. After falling to 7 million in 1400, by 1500 it was back to 10 million. In England, population dropped from 3.75 million to 2.5 million during the Black Death, and then by 1500 was back to 3.75 million.[4]

In sum, the simple Malthusian model provides a useful decription of how living standards could remain stagnant for long periods of time. The limited supply of land, combined with a positive relationship of income and population growth, leads to a situation where any increases in income per capita are inevitably temporary. It is important to note that $\underline{c}$ need not be equal to an absolute biological minimum level. If families have a value of $\underline{c}$ that is well above the biological minimum, then income will be stagnant, but at a relatively high level. The model does not necessarily mean that misery is "an absolutely necessary consequence," despite Malthus's predictions.

This model is a good place to begin, but clearly history shows us that we have not remained in a strict Malthusian equilibrium. In particular, the size of the human population had increased exponentially over time, while at the same time income per capita is no longer stagnant but increasing at a steady rate. To explain these two phenomenon, we will need to incorporate two elements into the baseline Malthusian model. The first is to allow for technological change. The second is to allow for the breakdown of the

[4] All the population figures are from McEvedy and Jones (1978).

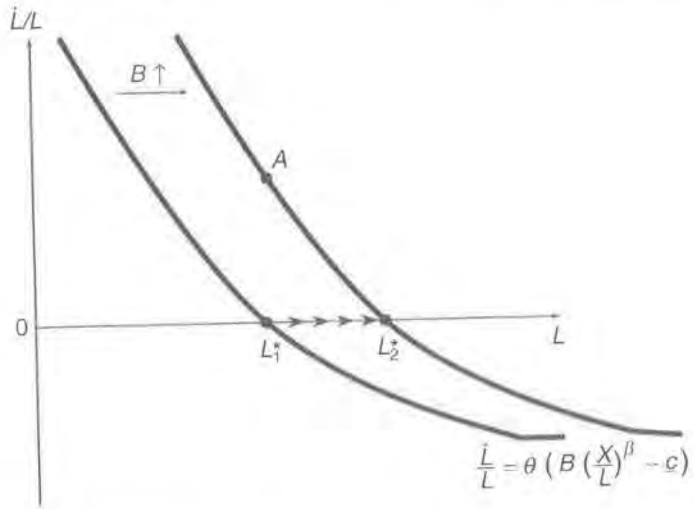
positive relationship between population growth and income per capita. Once we have those elements in place, we'll be able to provide a coherent explanation for the observed pattern of historical growth.

8.2.2 TECHNOLOGICAL CHANGE

To begin with, consider what happens if there is a one-time shift up in the technology term, B. This makes everyone more productive with the resources they have, raising income per capita and population growth as well. This can be seen directly in equation (8.4), where an increase in B raises $\dot{L}/L$ for any given level of population.

Figure 8.3 shows this graphically for an economy that begins with a steady-state population of L_1^*. The increase in B shifts the $\dot{L}/L$ curve to the right. Immediately after the shift, population is still L_1^*. However, with better technology these people earn a higher income per capita, which leads to an increase in population growth. The economy therefore

FIGURE 8.3 TECHNOLOGICAL CHANGE IN THE MALTHUSIAN MODEL



Note: When technology, B, increases this shifts the population growth curve to the right. Initialy, with population still at L_1^*, population growth jumps up to A. This allows the population to grow from the initial steady-state L_1^* to a larger steady-state size, L_2^*.

jumps to point A, with $\dot{L}/L > 0$. Population starts to increase, going from L_1^* toward L_2^*. As the population grows, income per capita declines, and population growth declines as well. Eventually, the economy comes to rest at the new steady-state level of L_2^*.

Note that this increase in population level is permanent. The increase in B has allowed the economy to support a greater number of people on the original amount of land. Technology does not share the rivalrous nature of land, so the increase in population does not eat away at the gains of technology. However, note that while population size is permanently higher, the level of income per capita will settle back down to $y^* = \underline{c}$. Technology in the Malthusian model leads to only temporary gains in living standards but permanent gains in population size. This is what we see going on in the Malthusian era, as living standards were stagnant for long stretches of time, but the absolute population of the Earth continued to increase.

8.2.3 CONTINUOUS TECHNOLOGICAL GROWTH

Rather than conceiving of technology as a set of exogenous shocks to B, we can consider continual growth in B. One can think of this as the population growth curve in Figure 8.3 shifting to the right repeatedly. This would allow for population size to grow continually, which implies that income per capita would have to be above y^*.

To see the effect of constant growth in B more clearly, take the production function involving land, take logs, and then the derivative with respect to time. This gives us

$$\frac{\dot{y}}{y} = \frac{\dot{B}}{B} - \beta\frac{\dot{L}}{L}, \tag{8.7}$$

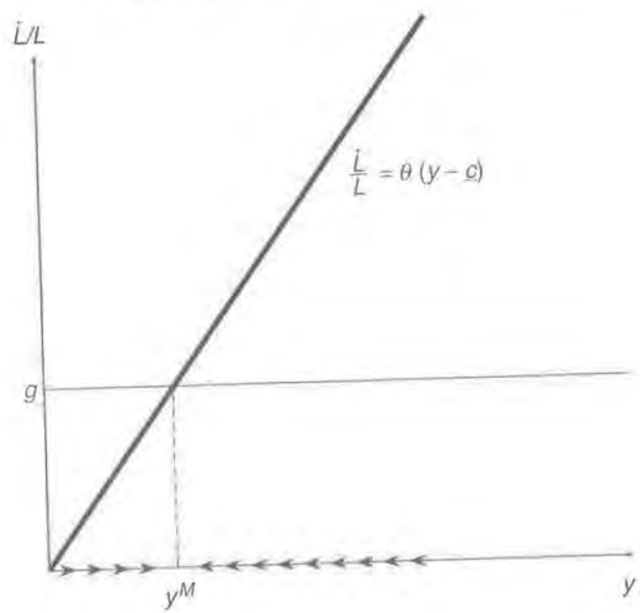
where we've explicity incorporated the fact that land is in fixed supply and the growth rate $\dot{X}/X = 0$. What we can see from this is that the growth of income per capita depends on how fast technology is growing relative to population.

Let $g = (1/\beta)\dot{B}/B$. Then if $\dot{L}/L < g$, income per capita is rising, as population growth is small relative to technology growth. If $\dot{L}/L > g$, then population growth is so large that income per capita is falling despite the technology growth. Finally, if $\dot{L}/L = g$, then the two effects balance out and income per capita is constant.

We can combine this with the standard Malthusian population growth equation (8.3) to analyze the dynamics of the model with constant technological progress. To do this, we're going to alter the type of diagram we're using, as this will set up the explanation of the transition to sustained growth. Figure 8.4 plots the population growth rate, $\dot{L}/L$, against income per capita, y, as opposed to the size of population as in Figure 8.2.

Give the dynamics of income per capita from equation (8.7), we know that if population growth is equal to g, then income per capita is constant. This can be seen as the point where the two lines intersect in the diagram. This happens at y^M, the Malthusian steady-state level of income per capita. If income per capita is below y^M, then population

FIGURE 8.4 DYNAMICS OF INCOME PER CAPITA WITH CONSTANT $\dot{B}/B$



Note: The figure shows the positive relationship between population growth and income per capita. From equation (8.7) we know that if population growth is above g income per capita is falling, and if it is below g income per capita is rising. The arrows on the x-axis show the dynamics of income per capita, and that in the long run the economy will always end up at y^M.

growth is smaller than g, and income per capita is growing. The opposite occurs if income per capita is higher than y^M, where population growth is larger than g and income per capita is shrinking.

What this means is that the income per capita of y^M is a stable steady state. Income will tend toward this level, regardless of where it starts. The steady-state income per capita is directly related to the growth rate of technology. If technology growth increases, this shifts up the horizontal line g, and the steady-state level of income per capita increases. Faster technological growth means that the force pushing up income per capita is getting stronger relative to the force pushing down income per capita—a large population.

Constant growth in technology in the Malthusian model does not lead to sustained growth in income per capita, though. It only increases the steady-state *level* of income per capita. It does, however, lead to sustained growth in population size. In steady state, it must be that $\dot{L}/L = g > 0$, so that the number of people is increasing. If the growth rate of technology is small, then population growth will be small, but so long as there is some technological progress, the population will grow. Mechanically, Figure 8.4 explains how we can have a growing population but stagnant income per capita, as observed for much of human history.

8.2.4 ENDOGENOUS TECHNOLOGICAL CHANGE

This brings us back to our original work on the sources of technological growth. What determines $\dot{B}/B$, and therefore g? The population size. Recall from the discussion in Chapter 5 that technological growth can be modeled as

$$\frac{\dot{B}}{B} = \nu \frac{s_R L^{\lambda}}{B^{1-\phi}}, \qquad (8.8)$$

which is increasing in the size of the population. Assuming that we start out historically with a very low level of L compared to B, then as population grows the growth rate $\dot{B}/B$ will increase.[5] Increases in $\dot{B}/B$

[5] To see this, consider that $\dot{B}/B$ will rise so long as $\lambda\dot{L}/L > (1-\phi)\dot{B}/B$. Plugging back in for $\dot{B}/B$, this will hold if $\lambda\dot{L}/L > \nu s_R \frac{L^{\lambda}}{B^{1-\phi}}$. For sufficiently small ratios of L to B, this will hold.

generate higher growth rates of population, though, which in turn generates higher technological growth, and so on and so on. This virtuous cycle will ultimately be the source of the transition to modern growth.

This reasoning is behind the work of Michael Kremer (1993), who uses it to explain the relationship of population growth and population size over human history. Kremer's model of technological change is a special case of equation (8.8) in his main analysis. He assumes that $\lambda = 1$, $\phi = 1$, and simplifies things by assuming that all people both work and create innovations, so that s_R drops out. We noted in Chapter 5 that the assumption of $\phi = 1$ was unrealistic given modern growth rates, but here we are trying to explain growth prior to the modern era. The assumption that $\phi = 1$ is not strictly necessary, but we do need a sufficiently strong "standing on shoulder" effect, implying that ϕ is large. Combining Kremer's assumptions with the Malthusian equilibrium condition that $\dot{L}/L = g$ gives us

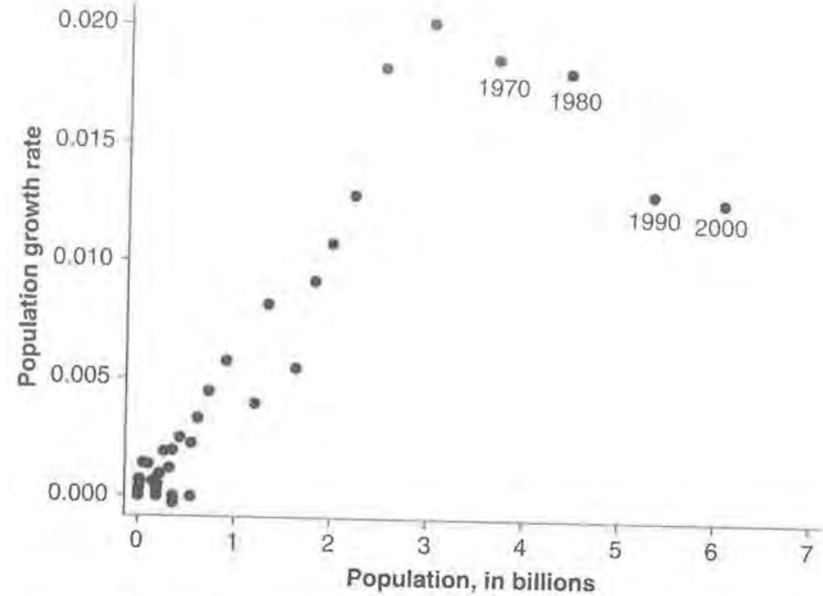
$$\frac{\dot{L}}{L} = \frac{\nu}{\beta}L. \tag{8.9}$$

In short, the population growth rate is increasing with the size of the population. This captures in very stark form the virtuous cycle described previously. As population size increases, technology grows faster. As technology grows faster, the population grows faster. The really intriguing thing about Kremer's model is that we can actually look at data on population growth and population size to see if it works.

Figure 8.5 plots the growth rate of population against the size of the population, for years ranging from 1 million BCE to the present. As can be seen, for all but the most recent past, there is a very strong positive relationship. From the very origins of humanity until the middle of the twentieth century, every time that population size increased so did the subsequent growth rate of the population. It is only since 1970 that this relationship has broken off.

Note that the standard Malthusian model without endogenous technological change cannot match this data. In the strict Malthusian model, any increase in population size would be associated with lower income per capita, and hence *lower* population growth rates, the exact opposite of what we see in Figure 8.5. It is only once we introduce technological progress that depends positively on population size that we can explain the data in the figure. Although Malthusian mechanisms

POPULATION GROWTH AND POPULATION SIZE, 1 MILLION BCE TO 2011



SOURCE: Authors' calculations using data from Kremer (1993) and U.S. Census Bureau data on world population.

may be at work in the world, they have consistently been overcome by the positive effects of population size on innovation.

The Kremer model is useful for describing nearly all of human history. However, in this model the virtuous cycle of population growth and technological change will end up spiraling into growth rates for both that are neither observed nor even believable. To see this, consider that in Figure 8.4, every time g increases population growth increases as well, and this can continue forever. That is, we should see accelerating growth rates of technology and accelerating growth rates of population over time. While there has certainly been some acceleration of both growth rates through the post-Malthusian era and into the modern era, the data in Figure 8.5 show that population growth is now falling. We'll need to incorporate more nuance into our description of population growth to accommodate these facts.

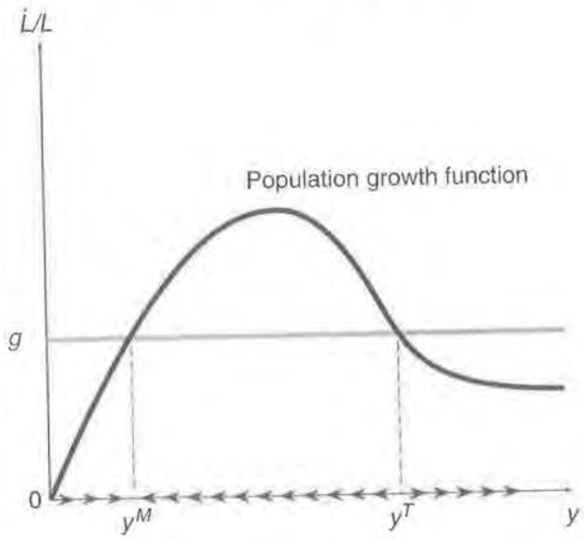
8.3 THE TRANSITION TO SUSTAINED GROWTH

The next element to add to the model is a description of population growth that does not increase continuously with income per capita. We'll describe the new population dynamics mechanically first, and show how with these mechanics in place we can provide a comprehensive description of growth from the deep past until today. The section following this one will describe the economics underlying these population dynamics.

8.3.1 REALISTIC POPULATION GROWTH RATES

Figure 8.6 shows our more refined function relating population growth to income per capita. At very low levels of income per capita, there is a positive relationship between population growth and y, as in the

FIGURE 8.6 DYNAMICS OF INCOME PER CAPITA



Note: The population growth function here captures the fact that there is a turning point at which increases in income actually lower population growth rates. With this function, there are now two steady states, y^M and y^T. For any $y < y^T$, income per capita will end up at y^M eventually, as in the standard Malthusian model. If $y > y^T$, then population growth is lower than g and income per capita will grow continuously.

standard Malthusian model. However, in Figure 8.6 there is a turning point at which population growth no longer rises with income per capita but actually starts to decline. Continued increases in income per capita lower the population growth rate until it levels out at very high income levels.

The dynamics of y itself are still governed by equation (8.7). Therefore, it is still the case that if $\dot{L}/L < g$, income per capita is rising as technology is improving faster than population is eating away at resources. For population growth rates above g, income per capita is falling. These dynamics are denoted on the x-axis in the figure.

What we end up with is two steady states for income per capita. The point y^M is a stable steady state, and income per capita will end up here as long as $y < y^T$ to begin with. All of our intuition from the Malthusian model holds here. Income per capita is stagnant at y^M. On the other hand, if income per capita is larger than y^T, then population growth is lower than g, and income per capita is increasing. Given the population growth function, this does not cause a Malthusian response of increasing fertility rates, and so growth in income per capita continues unabated. When $y > y^T$, it is the case that $\dot{L}/L < g$ forever, and growth in income per capita does not stop. y^T is a steady state, but an unstable one. If income is not equal to y^T, it will never end up at y^T.

This suggests one possible reason for the transition to sustained economic growth. It is possible, given Figure 8.6, that a sufficiently large shock in income per capita would allow an economy that was at y^M to jump to $y > y^T$. This would have been sufficient to put the world on the sustained growth trajectory. However, this possibility does not match the historical experience. Remember that during the Black Death income per capita more than doubled, and this was *not* sufficient to kick off sustained growth. So it is hard to see how an even larger jump in y could have taken place in the nineteenth century that we have somehow missed in the data.

Rather than jumping to sustained growth, the source of the transition lay in the steady increase in the size of population and the consequent increase in technological growth. Recall from the previous section that as population size increases, g rises, shifting up the horizontal line in Figure 8.6. This shifts y^M up over time, which shows up as minor increases in income per capita over the Malthusian era.

So long as technological growth is positive, population growth is positive as well, and this continues to accelerate technological growth.

The horizontal line denoting g moves up as the number of people—and therefore the number of innovators—increases. Given that the population growth function now has a maximum, it is possible for technological growth to increase to the point that g actually lies above the entire function.

This situation is shown in Figure 8.7. Once there is a sufficiently large population, the economy is "freed" from the Malthusian steady state. In Figure 8.7, it is *always* the case that $\dot{L}/L < g$. No matter the level of income per capita, growth in income per capita is always positive. Eventually, income per capita will increase to the point that population growth levels off at the rate n^*.

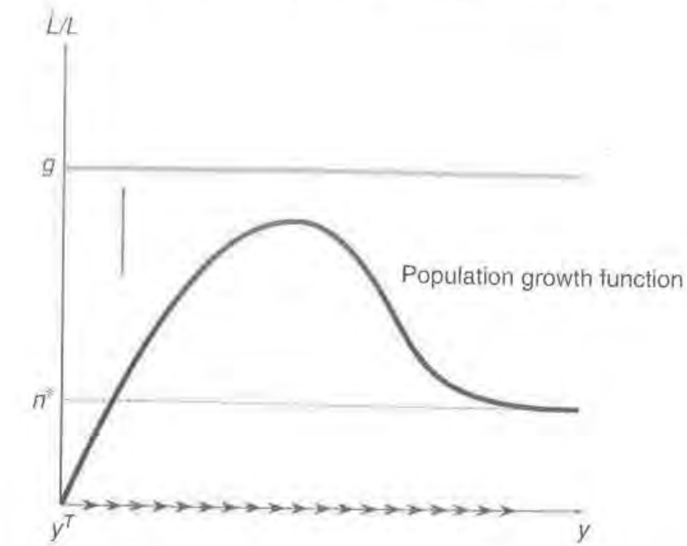
8.3.2 FROM MALTHUSIAN STAGNATION TO SUSTAINED GROWTH

With this, we have all the pieces to provide a description of the pattern of economic growth for all of human history. In the beginning, the population is very, very small, and exists in a Malthusian steady state. With the very small population, g is extraordinarily small given equation (8.8). Slow, almost imperceptible technological growth means slow growth of population size in the Malthusian steady state. This situation persists for thousands of years, with the economy inching ahead in the number of innovations and number of people.

Despite the slowness, there is an acceleration under way, with the cycle of increasing speed of innovation leading to increasing population size, and back to increasing speed of innovation. g is increasing, and with it y^M. The economy is still in a Malthusian environment, but the pace of everything is starting to speed up. In Western Europe, one might find this period starting around 1500 or 1600, whereas it took slightly longer in other areas of the world.

With increased population size, and technological growth speeding up, the economy enters what we called the post-Malthusian period. In terms of Figure 8.6, the economy is transversing the "hump" in the population growth function. Income per capita is rising slowly, as most of the gains in output are taken up by increasing population growth rates. However, the very fast population growth raises the growth rate of technology, and the world reaches a situation like that in Figure 8.7 where g is larger than the maximum rate of population growth. A demographic transition leading toward lower population growth sets in as income per capita continues to increase, and we reach income levels at which population growth settles down to the rate n^*.

FIGURE 8.7 THE TRANSITION TO SUSTAINED GROWTH



Note: The figure shows that once the growth rate of technology is high enough, the growth rate of income per capita is always positive. This means that the economy will grow continuously, and the population growth rate will eventually settle down to n^*.

In this final phase, with a constant population growth rate, our work from Chapter 2 through 5 becomes the best description of the world. Here, population size has become very large and the growth rate of technology stops accelerating. With constant growth in population, the world settles into a balanced growth path, and along this path we know that technology grows at

$$g_B = \frac{\lambda}{1-\phi}n^*, \tag{8.10}$$

Growth in income per capita, along the ultimate balance growth path, will be

$$g_y = g_B - \beta n^* = \left(\frac{\lambda}{1-\phi} - \beta\right)n^*, \tag{8.11}$$

given the production function including land. This is slightly different from what we found in the original models of growth in Chapters 2 through 5, and that is because of the inclusion of a fixed factor of production, land. This produces a drag on growth of βn^* as the available land is spread more and more thinly among the increasing population. So long as $g_B > \beta n^*$, growth in income per capita will be positive along the balanced growth path. This will be the case if $\lambda/(1-\phi) > \beta$. That is, if the "stepping on toes" effect is not too strong (i.e., λ is relatively large) and the "standing on shoulders" effect is not too weak (i.e., ϕ is not too a small), then growth will be positive. Additionally, if the role of land in production, β, is small, then growth will be positive, something we take up in the next section in more detail.

Ultimately, the pattern of growth in income per capita and population across human history can be explained as an outgrowth of an expanding population. As the number of potential innovators has increased over time, so has the speed at which technological progress occurs, and ultimately this allowed the world to leave Malthusian stagnation behind and embark on the path of sustained economic growth that we experience today.[6]

8.3.3 STRUCTURAL CHANGE DURING THE TRANSITION TO GROWTH

The previous analysis established that an important aspect of the takeoff to growth was having population size sufficient to get g larger than even the highest possible population growth rate, as in Figure 8.7.

Recall that $g = g_B/\beta$. We have discussed the model in terms of technological growth, implying that what is happening is that g_B eventually rises sufficiently to allow the escape from the Malthusian era. It is also worth considering the role of β.

The larger is β, the more land matters for production, and the smaller is g. Thus, making the fixed factor of production more important will delay the time at which we transition out of the Malthusian era. On the other hand, decreases in β will raise g, and this will make it easier to escape. In the limit, as β goes to zero, g goes to infinity, and the economy can escape the Malthusian era immediately, even with a very

[6]The description in this section is stylized, and one can provide a more rigorous mathematical description of the dynamics involved. See Galor (2011) for a comprehensive treatment of the subject.

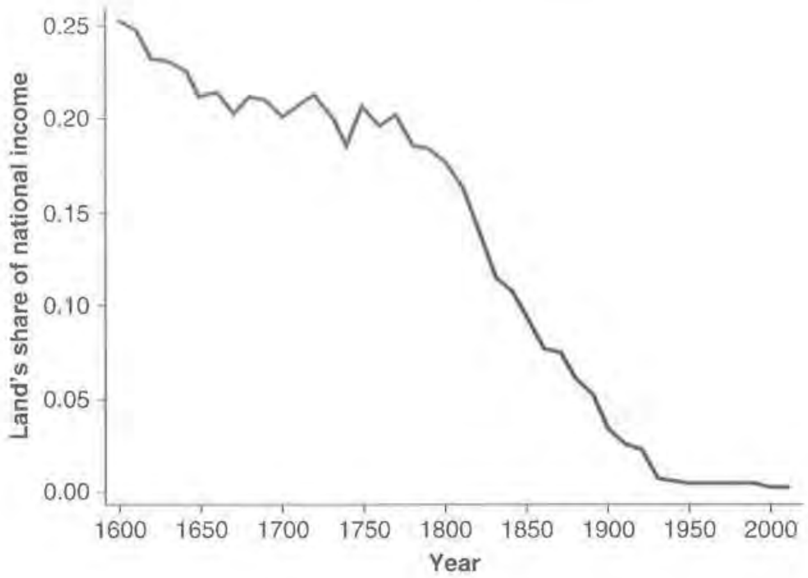
small population. This demonstrates how crucial having a fixed factor of production is to delivering the Malthusian results.

It also shows that structural changes in the economy may have played a role in reaching sustained growth. Agricultural goods made up the vast majority of total output early in the Malthusian era. As the economy developed, many of the innovations that occurred led to new products being introduced that made the contribution of fixed factors like land less important. For example, even though cotton is an agricultural good requiring land to grow, a great deal of the final value of a cotton shirt can be attributed to the skilled work involved in creating thread from the raw cotton, weaving the cotton thread into cloth, and sewing the raw cloth into a wearable shirt. On top of those direct production processes, there is the value added by people who transport the shirt from factory to store, the clerks who stock the shirt and direct you to it on the shelf, and to the designer who came up with a style of shirt that you want to buy. In the end, the land involved in producing the raw cotton, while necessary, captures only a very small portion of the value of the shirt.

Over time, then, the evidence is that land's share in output, captured by β, is declining. This is demonstrated clearly in Figure 8.8 for England. Around 1750, farmland rents made up 20 percent of national income in England, whereas by 1850 this had fallen to about 8 percent, and it was less than 0.1 percent in 2010. So at the same time that technological growth was accelerating due to larger populations, the drag on the economy due to the fixed factor, land, was declining. This contributed to the increase in g that released us from Malthusian era.

If the limits imposed by a fixed factor like land are so pernicious, then why did past generations not focus on production that was not dependent on it? One reason was that the necessary innovations may not have been available yet. Another, and likely very relevant reason, was that past generations could not abandon agricultural production without starving to death. If we rank goods in terms of their importance, food is likely to come in at first place. In addition, there is some minimum quantity of food that would have been necessary just to keep people alive. So first and foremost, economies in the past would have had to allocate their labor to agricultural production until a sufficient amount of food was available. Only then could they turn to nonfood production that did not depend so crucially on a fixed factor like land.

LAND'S SHARE OF INCOME, ENGLAND, 1600–2010



SOURCE: Authors' calculations using data from Clark (2009).

However, while over history people have purchased more food as their incomes increased, the additional food purchases never appear to grow as quickly as income. To be more succinct, the income elasticity of food is less than one, something referred to as Engel's law (after Ernst Engel, a Prussian statistician of the nineteenth century). This means that as economies grew richer, a smaller fraction of their output was agricultural, and a smaller fraction of the labor force was engaged in the agricultural sector. Exact data from the distant past are not available, but in 1785 England already had only 40 percent of output made up of agricultural goods, falling to 5 percent by 1905. In Germany the fraction fell from close to 50 percent in 1850 to about 25 percent by 1905 (Mitchell, 1975). In contemporary times, the least developed parts of Sub-Saharan Africa produce about 50 percent of total output as agricultural goods, whereas in the United States this is less than 1 percent.

The structural transformation of economies from agricultural to producing mainly manufacturing goods or services went hand in hand

with their escape from the Malthusian era. This transformation would have contributed to the takeoff by reducing the role of land in production, and shrinking β. However, the underlying driver of the transition to sustained growth remains the increase in technological growth rates associated with larger populations, without which the income gains leading to the structural transformation would not have taken place.

THE ECONOMICS OF POPULATION GROWTH

We have described how, given the population growth function in Figures 8.6 and 8.7, we can completely describe the path of income per capita over time. Here we set out to describe the economics behind those figures. Why does population growth rise and then fall with income? In answering this question, we'll also provide an explanation for why education does not rise until population growth spikes.

We'll think about children and their education as goods that families "consume." This means that we can use the standard microeconomic framework of utility functions and budget constraints to describe the decisions that families make. The origins of the economic analysis of family behavior lie with Nobel-laureate Gary Becker (1960). He was focused on the opportunity cost of children, and explained declining fertility rates in developing countries as reflecting the increased cost of children to parents who were experiencing rising wages.

Becker's (1960) original theory regarding fertility, however, is not sufficient for explaining the broad shifts in population processes over history. It suggests that populations growth is highest when wages are lowest, and that is counterfactual given the evidence of the Malthusian period.

We instead adopt a "quantity/quality" framework. This adapts Becker's (1960) original work to say that parents care not only about how many children they have (the quantity) but also about their quality. Quality can mean many things, but we will measure it by the education that a child receives. Most importantly, the quantity/quality framework says that there is a tradeoff between the two, and that families can either have large families with low education, or small families with high education, but cannot afford to have many highly educated children. This quantity/quality tradeoff is featured in the original unified growth model of Galor and Weil (2000), and is capable of explaining what we see in Figure 8.4.

The quantity/quality theory is an economic description of family decisions regarding the number of children and the education they receive. As such, we will have both a budget constraint as well as a utility function. What is the budget constraint of a family? We'll examine an average family, earning exactly income per capita, y.[7] That income is spent having children, educating children, and providing a subsistence consumption for the parents. You can think of this subsistence consumption as the food and goods that parents require to maintain their own lives, and it serves the same purpose as in our original Malthusian model. We could allow for parents to increase their consumption as their income grows, but that would not meaningfully change any of our results. The budget can be written as

$$y = \underline{c} + M + E, \tag{8.12}$$

where $\underline{c}$ is the subsistence consumption amount, M is the amount spent on children, and E is the amount spent educating children. One can think of M as resources, such as food and clothing, that are necessary for each child, and E as additional resources spent on optional things such as formal schooling.

To translate spending on children and education into numbers of children and units of education, we'll use the following equations:

$$m = \eta \frac{M}{y} \tag{8.13}$$

$$u = E + \overline{u}. \tag{8.14}$$

The first says that the number of kids, m, depends positively on the amount of resources spent having children (M), but that this is offset by the level of income per capita y. What we are saying here is that the "price" of children is rising along with income. Why? This is capturing Becker's original idea that children take up a lot of parents' time, and as y rises their time is more and more valuable. The value η is a parameter that will help determine the long-run population growth rate.

The second equation describes the units of education of a child, u, which you may recognize as the input to the formula for human capital in earlier chapters. It is the sum of spending, E, and a separate term $\overline{u}$

[7] A family may well consist of two adults, both working and earning a total of $2y$. It's not crucial whether we use y or $2y$ as the income level of a family.

that captures an inherent amount of education that children get even if they are not formally schooled. You can think of $\overline{u}$ as representing the basic skills that any child will acquire by interacting with their family and community.[8]

We now have the constraints in place, so we have to specify the utility function of parents. We'll assume that it takes the following form,

$$V = \ln m + \ln u, \tag{8.15}$$

where V is their total utility. What this says is that parents derive utility from both the number of kids they have, m, and the amount of education that each child has, u. We presume that parents get no utility from their subsistence consumption $\underline{c}$, which is not crucial to what we are trying to show. The natural log function means that parents have diminishing marginal utility from both m and u. For example, the gain in utility going from one to two kids is bigger than the gain in utility going from two to three. The utility function also implies that parents will always want a positive number of both kids and education, as if either went toward zero, utility would drop to negative infinity!

To solve for the optimal choice of the family, we will incorporate the budget constraints into the utility function. First, use the relationships in equations (8.13) and (8.14) to write the utility function as

$$V = \ln \left(\eta \frac{M}{y} \right) + \ln (E + \overline{u}).$$

Then, to get things only in terms of education, use the spending constraint in equation (8.12) to substitute for M so that we have

$$V = \ln \left(\eta \frac{y - \underline{c} - E}{y} \right) + \ln (E + \overline{u}). \tag{8.16}$$

This looks somewhat convoluted, but notice that the only choice variable left is E, the amount to spend on education. We can now maximize utility with respect to E, finding out the optimal amount of education that parents will choose. Following that, we can use the budget constraint to find out the number of children.

[8] Note that E is total education spending by the family, but each child benefits equally from this. This assumption is not crucial but makes the analysis clearer.

To get the first-order condition for the maximization, take the derivative of equation (8.16) with respect to E and set it equal to zero,

$$\frac{-1}{y - \underline{c} - E} + \frac{1}{E + \bar{u}} = 0.$$

Rearranging this first-order condition to solve for E, we find

$$E = \frac{y - \bar{u} - \underline{c}}{2}.$$

This tell us that families will increase their spending on education as income increases. However, notice that if income is particularly small, then the solution implies that education spending could actually be negative. This doesn't make any sense, as the minimum that parents can spend on education is zero.

We'll have to be more careful in how we describe the optimal solution, taking into account the minimum for education spending,

$$E = 0 \qquad \text{if } y < \underline{c} + \bar{u}$$

$$E = \frac{y - \bar{u} - \underline{c}}{2} \qquad \text{if } y \geq \underline{c} + \bar{u}.$$

For low levels of income per capita, parents will not provide any education funding at all. Why? Recall that kids will always have at least $\bar{u}$ in education, so parents get that for "free." When income is very low, it makes sense to spend all of your money on having extra children rather than adding extra education to each child.

Knowing the solution for E we can use equation (8.12) to solve for M, and then use (8.13) to solve for m. If we put all this together, we'll find that

$$m = \eta\left(1 - \frac{\underline{c}}{y}\right) \qquad \text{if } y < \underline{c} + \bar{u}$$

$$m = \frac{\eta}{2}\left(1 - \frac{\underline{c}}{y} + \frac{\bar{u}}{y}\right) \quad \text{if } y \geq \underline{c} + \bar{u}$$

is the solution for the number of children.[9]

[9]The population growth rate can be calculated directly from the number of children. Assuming that each family hast two adults in it, then population growth, n, can be written as $n = \frac{m-2}{2}$.

Here it is worth recalling what is the objective of this section. We are trying to describe the population growth function in Figures 8.6 and 8.7. Specifically, why does population growth *rise* with income when incomes are low (the upward sloping part of the function) and the *fall* with income at higher income levels (the downward sloping part of the function)? Our solution for families' optimal number of children, m, delivers these relationships.

To see this, consider first the case when income is low and $y < \underline{c} + \bar{u}$. Here, if income increases, so does the optimal number of children, $\eta(1 - \underline{c}/y)$. When families are relatively poor, an increase in income leaves more resources left over after paying for subsistence consumption $\underline{c}$, and parents are able to afford to have more children. Recall that at this low level of income, parents will not spend any money educating their children. Our model therefore offers a description of the Malthusian-era population dynamics: population growth is positively related to income and education levels are minimal.

What happens when income reaches $y = \underline{c} + \bar{u}$? At this point, which represents the peak of the population growth function in Figures 8.6 and 8.7, parents begin to start investing in their children's education. In addition, assuming that $\underline{c} < \bar{u}$, then as income increases further the optimal number of children will fall.[10] As y continues to go up, the "price" of children rises along with it, and n decreases, giving us the downward sloping portion of the population growth function. This represents the demographic transition, with the tendency toward smaller families and greater education.

In the long run, as y gets very large, the terms $\underline{c}/y$ and $\bar{u}/y$ both approach zero, and m approaches $\eta/2$. The value $\eta/2$ dictates the long-run population growth rate, which no longer depends on income.[11] The model in this section therefore provides an explanation for the long-run population growth rate used in our models of technological change from Chapter 5. For countries that have sufficiently high income levels, the population growth rate is predicted to remain unchanged even as income per capita continues to increase.

[10]$\underline{c} < \bar{u}$ is necessary to produce the results, but as these are very stylized terms there is no way to link them directly to data proving the condition holds.

[11]Specifically, the long-run population growth rate, n^* can be written as $n^* = \eta/4 - 1$. Note that nothing requires n^* to equal zero, and population could well continue to increase indefinitely.

The quality/quantity model of family choices regarding children and education is therefore able to provide a justification for the population growth function plotted in Figures 8.6 and 8.7. Combined with endogenous technological change, we can provide an explanation for why humans remained mired in the Malthusian era a very long time before eventually transitioning into the current world of sustained economic growth as population size increased.

8.5 COMPARATIVE DEVELOPMENT

The model laid out in this chapter is able to provide an explanation for the observed pattern of income per capita and population growth across the whole world. Is there anything we can say, using this model, about disparities in living standards? In particular, what does this model imply about why it was England (with the rest of Western Europe right behind them) that was the first place to make the transition to sustained growth? This is often tangled up with the question of why China was *not* the first place to make the transition.

Mechanically, achieving sustained growth requires that g rise above the population growth function. This can occur either because g rises (through faster technological progress or larger population size) or because the population growth function shifts down (through differences in family fertility behavior).

A number of authors, with David Landes (1969) and Joel Mokyr (1990, 2002) being among the most prominent, connect the early takeoff in Europe to technological creativity. One element of this creativity, particularly in England, appeared to be the willingness to borrow (or perhaps steal) ideas from other countries and regions, including China. Imitation would have allowed England to avoid duplication of research efforts, which in the model may be reflected as a less severe "stepping on toes" effect, implying a larger value for λ. Alternatively, the establishment of secure intellectual property rights in Europe, which we have discussed before in connection with the work of Douglass North (1981), would imply greater incentives to pursue innovation. Mechanically, we can think of this as introducing a higher level of s_R in Europe compared to other areas. Regardless, being able to sustain higher growth rates in technology would have allowed Europe to escape the Malthusian equilibrium sooner.

This explanation highlights the importance of *growth* in technology versus the *level* of technology. China historically developed any number of technologies before the Europeans. China had advanced metallurgy and was making steel several centuries ahead of Europe. In textiles production, China had early versions of multiple spinning devices and mechanical looms about two hundred years before the English would turn these into an engine of the Industrial Revolution. Paper, gunpowder, and mechanical clocks were all invented in China well before they appeared in Europe. Despite this very high level of technology, innovation was apparently not occurring fast enough to overcome the drag of population growth, and China remained locked in a Malthusian world.

Instead of faster technological growth, Europe's advantage may have been lower population growth rates. There are several reasons proposed for this difference with Asia. Max Weber (1920) speculated that Protestantism was integral in changing preferences for children's education, lowering fertility rates in northwest Europe. John Hajnal (1965) proposed that a distinct European marriage pattern, with relatively late ages of marriage and large number of unmarried women in the population, limited population growth. Looking at population processes from the other side, Voigtländer and Voth (2010) show that it may have been higher mortality rates in Europe—due to war, plague, and urbanization—that lowered population growth and ironically let Europe escape the Malthusian equilibrium sooner.

Geography may have played a role as well. Differences in the type of agriculture practiced could have led to a relatively high cost of children in Europe and lower population growth rates, as described by Vollrath (2011). More broadly, geography may have been a determinant of why it was Europe and Asia, as opposed to Africa or the Americas, that were the leading candidates to jump to sustained growth. Jared Diamond (1997) documents advantages in terms of domesticable crops and livestock for both Europe and southeast Asia. This functioned as an initial advantage in the level of technology, B in our model, which allowed these areas to sustain larger populations. With larger populations, technological growth was able to advance more quickly compared to less favored places such as Africa and the Americas.

SUMMARY

Population growth plays a central role in the process of economic growth. It not only determines the long-run growth rate, as seen in Chapter 5 but was crucial in releasing the world from Malthusian stagnation around 1800. To explain this transition we incorporated a fixed natural resource, land, into our model. The presence of this fixed factor means that larger populations tend to drag down living standards. However, as we saw when endogenous technological change is included, larger populations also mean greater rates of innovation that tend to push up living standards. For much of history the downward drag was more powerful and income per capita remained stagnant at relatively low levels. Eventually, though, the world population grew sufficiently large that innovation took place fast enough to overcome this downward drag and put us on the path to sustained growth.

The microeconomics behind family choices about fertility and education provide us with a way of understanding why population growth does not continue to increase with income per capita. Once families are rich enough they begin to invest in their children and further gains in income result in greater education but not higher fertility. This model of population growth helps us to understand some of the theories regarding comparative development across the world. A combination of a rapid rate of innovation and a low peak rate of population growth helped Europe become the first area of the globe to achieve sustained growth in income per capita, an advantage that it and offshoots such as the United States have maintained to the present day.

EXERCISES

1. *The Black Death.* In Section 8.2.1 we discussed how a major drop in the size of the population could actually raise living standards. Consider an economy that is described by the model in that section and is currently at the steady state level of population L^*.

   (a) There is a one-time drop in population, to $L^0 < L^*$, following an outbreak of the plague. Draw a graph showing the path of income per capita, y, in this economy over time. Include on the graph the time period prior to the plague, the plague itself, and the time period following the plague as the economy recovers back to steady state.

   (b) An alternative way to think of a plague is as a drop in the productivity of the population. Start over with an economy at the steady-state level of population L^*. Now let there be a permanent drop in productivity, B, due to the plague. Draw a new graph showing the path of income per capita, y, in this economy over time. Again include the period prior to the productivity drop, the drop itself, and the time period following as the economy goes to its steady state. How does income per capita compare in this situation to the one in (a)?

   (c) Finally, start over again with the economy at steady state with L^* in population. Now let there be a temporary drop in productivity, B, due to the plague. That is, B falls for several years, and then goes back to its original level. Draw a new graph showing how income per capita evolves over time, similar to the prior parts in this question. How does income per capita compare in this situation to that in (a) and (b)?

2. *The importance of growth rates versus productivity levels.* Consider two economies, A and B. Both economies are described by the model in Section 8.3, having a population growth function similar in form to that in Figure 8.6. You know that the population growth function's peak is at $\dot{L}/L = 0.02$, or 2% per year. Both economies start with a productivity level of $B = 1$. In economy A, productivity grows at 0.5% per year for 1000 years. In economy B, productivity is stagnant at $B = 1$ for 800 years. Then, for the next 200 years productivity grows at the rate 2.5% per year.

   (a) In the year 800, how much larger is productivity in economy A than in economy B?

   (b) In the year 1000, how much larger is productivity in economy A than in economy B?

   (c) Given what you know about the population growth function, will economy A ever take-off to sustained growth in income per capita? Will economy B be able to transition to sustained growth?

3. *Changes in basic education.* Assume there is an increase in the basic skills, $\bar{u}$, that each child receives. This may be because of the introduction of universal primary schooling, for example. In the quantity/quality model in Section 8.4, what effect does this have on the amount of education spending, E, that parents do? Does this affect the peak fertility rate? Does this affect the ability of a country to transition to sustained growth?

4. *A changing land share.*[12] In Section 8.3.3 we mentioned the role of structural transformation in contributing to sustained growth. Consider an economy that produces two goods, an agricultural good and a manufacturing good. An amount Y_A of the agricultural good can be produced using land and labor according to

$$Y_A = X^{\beta}(AL_A)^{1-\beta}, \tag{1}$$

where $\beta < 1$. An amount Y_m of the manufacturing good can be produced using labor only; no land is required:

$$Y_M = AL_M. \tag{2}$$

Assume that both these production functions benefit from the same technological progress, A. Finally, the economy faces a resource constraint for labor, $L_A + L_M = L$. For simplicity, assume that the price of the agricultural good in terms of the manufacturing good is one, so that total GDP in the economy is $Y = Y_A + Y_M$.

(a) Define $s = L_A/L$ as the fraction of the economy's labor force that works in agriculture. Assume that A and L are constants. What is total GDP in the economy, as a function of the allocation variable s and the exogenous parameters β, X, A, L?

(b) Find the allocation s^* that maximizes total GDP.

(c) What happens to s^* if A and L increase over time?

(d) Let the price of land P_x be given by the value of its marginal product. What happens to land's share of GDP, $P_x X/Y$, if A and L increase over time?

[12] This problem is inspired by Hansen and Prescott (1998).

9 ALTERNATIVE THEORIES OF ENDOGENOUS GROWTH

In the preceding eight chapters, we have laid out the basic questions of economic growth and some of the main answers provided by economic research. The next two chapters depart from this flow in two important directions. This chapter examines alternative theories of endogenous growth that have been proposed; in this sense, it could be read immediately following Chapter 5 or even Chapter 3. Chapter 10 turns to a question that has received much attention in the history of economic thought: the sustainability of long-run growth in the presence of finite natural resources. It, too, could be read any time after Chapter 3.

In this book, we have purposely limited ourselves to a few closely related models in an effort to formulate a general theory of growth and development. One result of this method of exposition is that we have not been able to discuss a large number of the growth models that have been developed in the last twenty-five years. This chapter presents a brief discussion of some of these other models.