Resources and Growth

Types of natural resources

- Renewable: land, trees. Can think of them as a constant stock. Know that presence will slow down steady state growth
- Non-renewable: oil, copper, iron. Will eventually run out. How do these alter steady state growth?

Let output be

$$Y = BK^{\alpha}E^{\gamma}L^{1-\alpha-\gamma} \tag{1}$$

where E is energy used in production. Other aspects of model will be typical

$$\frac{\dot{B}}{B} = g \tag{2}$$

$$\frac{\dot{L}}{L} = n \tag{3}$$

$$\dot{K} = sY - \delta K \tag{4}$$

Resources

Let R_0 be the initial stock of the non-renewable resource. With E used in production at any given time, we have

$$\dot{R} = -E \tag{5}$$

What is E? Let's assume that at any given moment, we consume a fixed fraction of the remaining stock of resources, so

$$E = s_E R \tag{6}$$

Turns out this is the result if we think about optimal models of resource use. So we'll stick with constant s_E.

Combined with top equation we have

$$\frac{\dot{R}}{R} = -s_E. \tag{7}$$

Evoluion of Resources

Given $\dot{R}/R = -s_E$ is a differential equation we could solve for the following

$$R(t) = R_0 e^{-s_E t} \tag{8}$$

and therefore

$$E(t) = s_E R_0 e^{-s_E t}. \tag{9}$$

This equation implies (take logs and derivatives) that

$$\frac{\dot{E}}{E} = -s_E \tag{10}$$

which implies that the amount of energy is declining constantly over time.

Note also that s_E has two effects on the amount of energy we use. The higher is s_E, the more of the resource we use right now, so E is higher. But the higher is s_E, the smaller the resource base left, so E will be lower over time.

Balanced Growth Path

Again, we want to find a BGP where output per worker grows at a constant rate, and all the terms in the production function grow at constant rates as well.

A little difficult given the E, so different strategy. With $Y = BK^{\alpha}E^{\gamma}L^{1-\alpha-\gamma}$ divide both side by Y^{α} to get

$$Y^{1-\alpha} = B\left(\frac{K}{Y}\right)^{\alpha} E^{\gamma}L^{1-\alpha-\gamma} \tag{11}$$

and then take both sides to $1/(1-\alpha)$ to get

$$Y = B^{1/(1-\alpha)}\left(\frac{K}{Y}\right)^{\alpha/(1-\alpha)} E^{\gamma/(1-\alpha)}L^{(1-\alpha-\gamma)/(1-\alpha)} \tag{12}$$

and in per-worker terms we have

$$y = B^{1/(1-\alpha)}\left(\frac{K}{Y}\right)^{\alpha/(1-\alpha)} E^{\gamma/(1-\alpha)}L^{\gamma/(1-\alpha)} \tag{13}$$

Solving for BGP

Given

$$y = B^{1/(1-\alpha)} \left(\frac{K}{Y}\right)^{\alpha/(1-\alpha)} E^{\gamma/(1-\alpha)} L^{\gamma/(1-\alpha)} \tag{14}$$

we can plug in what we know about E to get

$$y = B^{1/(1-\alpha)} \left(\frac{K}{Y}\right)^{\alpha/(1-\alpha)} \left(s_E R_0 e^{-s_E t}\right)^{\gamma/(1-\alpha)} L^{\gamma/(1-\alpha)}. \tag{15}$$

Now, take logs and derivatives to find

$$\frac{\dot{y}}{y} = \frac{1}{1-\alpha}\frac{\dot{B}}{B} + \frac{\gamma}{1-\alpha}\frac{\dot{E}}{E} - \frac{\gamma}{1-\alpha}\frac{\dot{L}}{L} \tag{16}$$

which accounts for the fact that along the BGP K/Y will be unchanging.

Growth along the BGP

Using

$$\frac{\dot{y}}{y} = \frac{1}{1-\alpha}\frac{\dot{B}}{B} - \frac{\gamma}{1-\alpha}\frac{\dot{E}}{E} - \frac{\gamma}{1-\alpha}\frac{\dot{L}}{L} \tag{17}$$

and what we already know we have that

$$g_y^{BGP} = \frac{1}{1-\alpha}g - \frac{\gamma}{1-\alpha}s_E - \frac{\gamma}{1-\alpha}n \tag{18}$$

or

$$g_y^{BGP} = \frac{1}{1-\alpha}\left(g - \gamma(s_E + n)\right) \tag{19}$$

So having non-renewables in acts something like the Malthusian model. We have

- Slower growth due to n. With a fixed resource, population growth puts a drag on growth in output per worker
- Because the resource is winding down, we have an additional drag on growth due to s_E
- Growth in y is positive only if $g > s_E + n$, or technological change is sufficiently fast.

Drag Effect

How large is the drag on growth from resources? First, let's allow for land as well,

$$Y = BK^{\alpha}X^{\beta}E^{\gamma}L^{1-\alpha-\beta-\gamma} \tag{20}$$

and X does not change over time. Solving this looks identical to what we did, but with β term carried through,

$$g_y^{BGP} = \frac{1}{1-\alpha}\left(g - \gamma s_E - (\beta+\gamma)n\right) \tag{21}$$

and n has an additional negative effect due to land.

Drag Effect

So what are these values? Nordhaus (1992) finds

- $\beta = 0.1$, $\gamma = 0.1$, and $\alpha = 0.2$
- $n = 0.01$
- $s_E = 0.005$, or we are using 1/2 of 1% of resources every year
- No, s_E probably isn't higher today. New discoveries of many resources mean that R_0 is actually going up.

The drag term

$$\frac{1}{1-\alpha}\left(\gamma s_E - (\beta+\gamma)n\right) = \frac{1}{1-.2}(.1 \times 0.005 + (.1 + .1) \times 0.01) = 0.003125 \tag{22}$$

or growth is about 0.3 of one percent lower per year because of resources.

Given that y grows at about 1.8% per year, this isn't insignificant. Growth could be 2.1% per year without effect of resources.

Scarce Resources

We seem to be running down the resources. Shouldn't they be getting more expensive? Think of factor shares for resources

$$v_E = \frac{P_E E}{Y} \tag{23}$$

and similarly let

$$v_L = \frac{wL}{Y}. \tag{24}$$

Then we could take ratio as

$$\frac{v_E}{v_L} = \frac{P_E E}{wL} \tag{25}$$

and rearrange to

$$\frac{P_E}{w} = \frac{v_E/v_L}{E/L} \tag{26}$$

or the ratio of energy prices to wages depends on relative factor shares (v_E/v_L) and energy per worker (E/L).

Energy Prices

The price of energy relative to wages

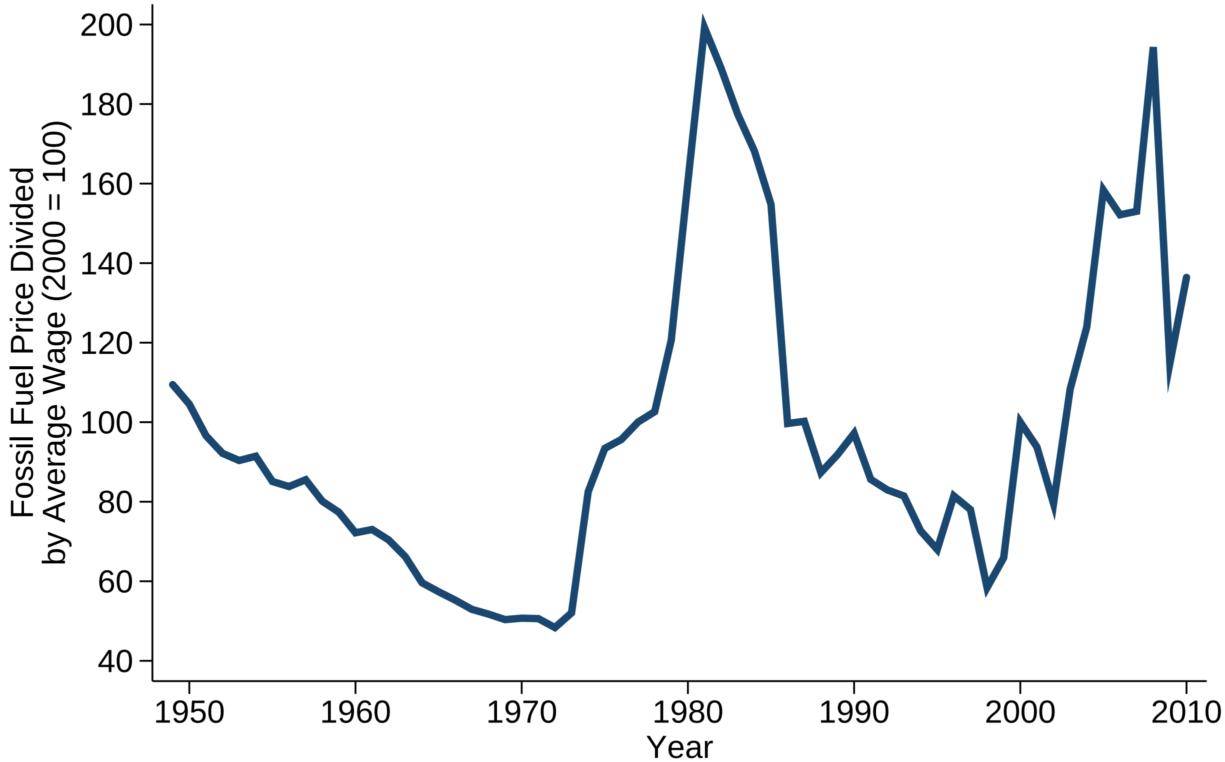
$$\frac{P_E}{w} = \frac{v_E/v_L}{E/L} \tag{27}$$

depends on E/L

- We would expect E to fall as we use resources, and L to rise with population
- So we would expect P_E/w to go up over the long run
- Assumes that v_E/v_L are roughly constant over time - are they?

Fossil Fuels and Wages

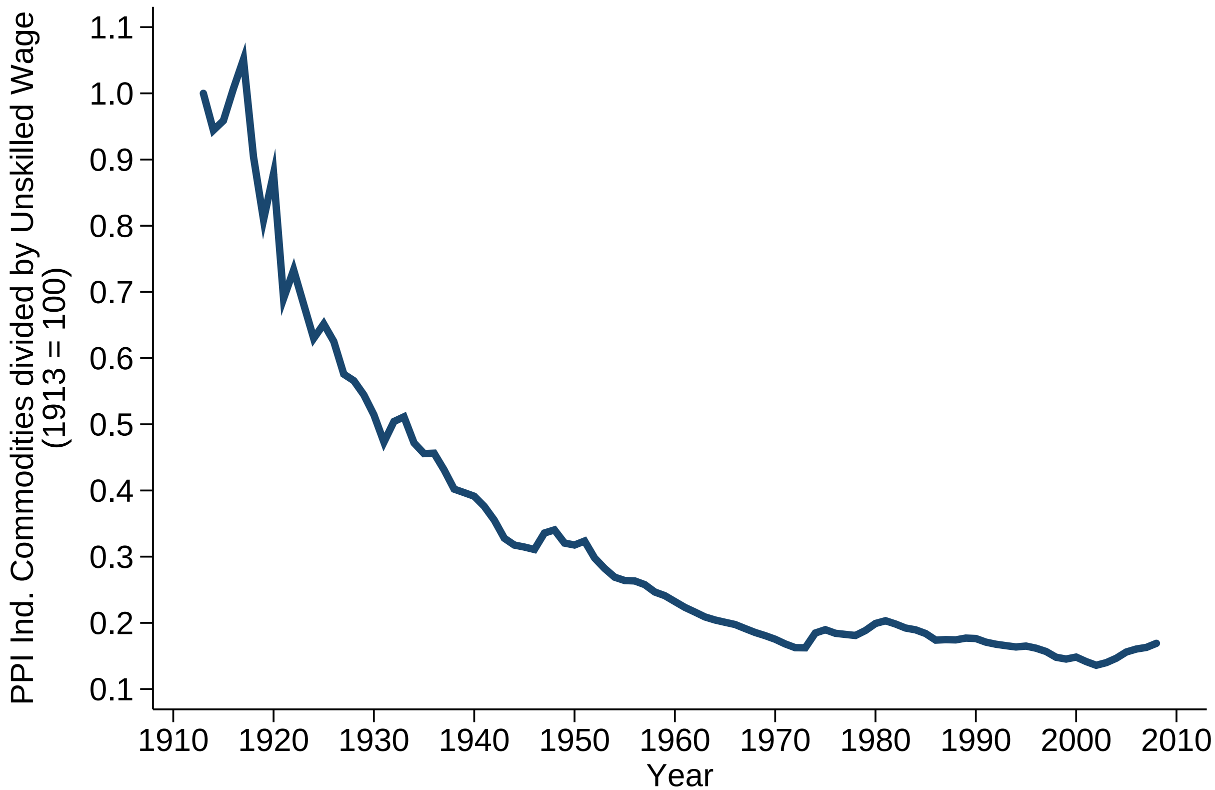
Is there a distinct upward trend?



SOURCE: U.S. Energy Information Administration (2012), Table 3.1 and U.S. Bureau of Labor Statistics series CES0500000008 for hourly earnings 1964−2010 and series EEU00500005 for hourly earnings 1949−1963.

Commodity Prices and Wages

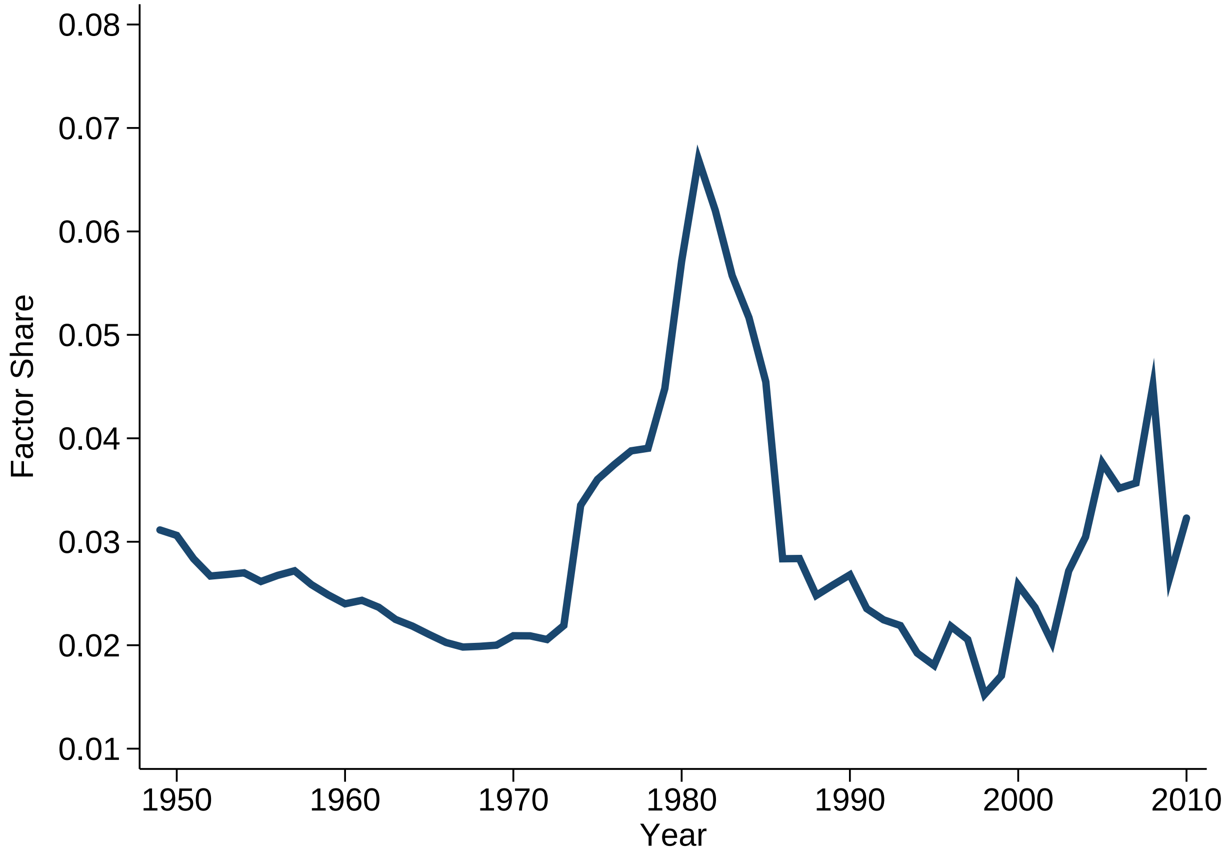
Definitive downward trend



Source: PPI Industrial Commodities index from the Bureau of Labor Statistics, series WPU03THRU15. Unskilled wages are from Williamson (2009).

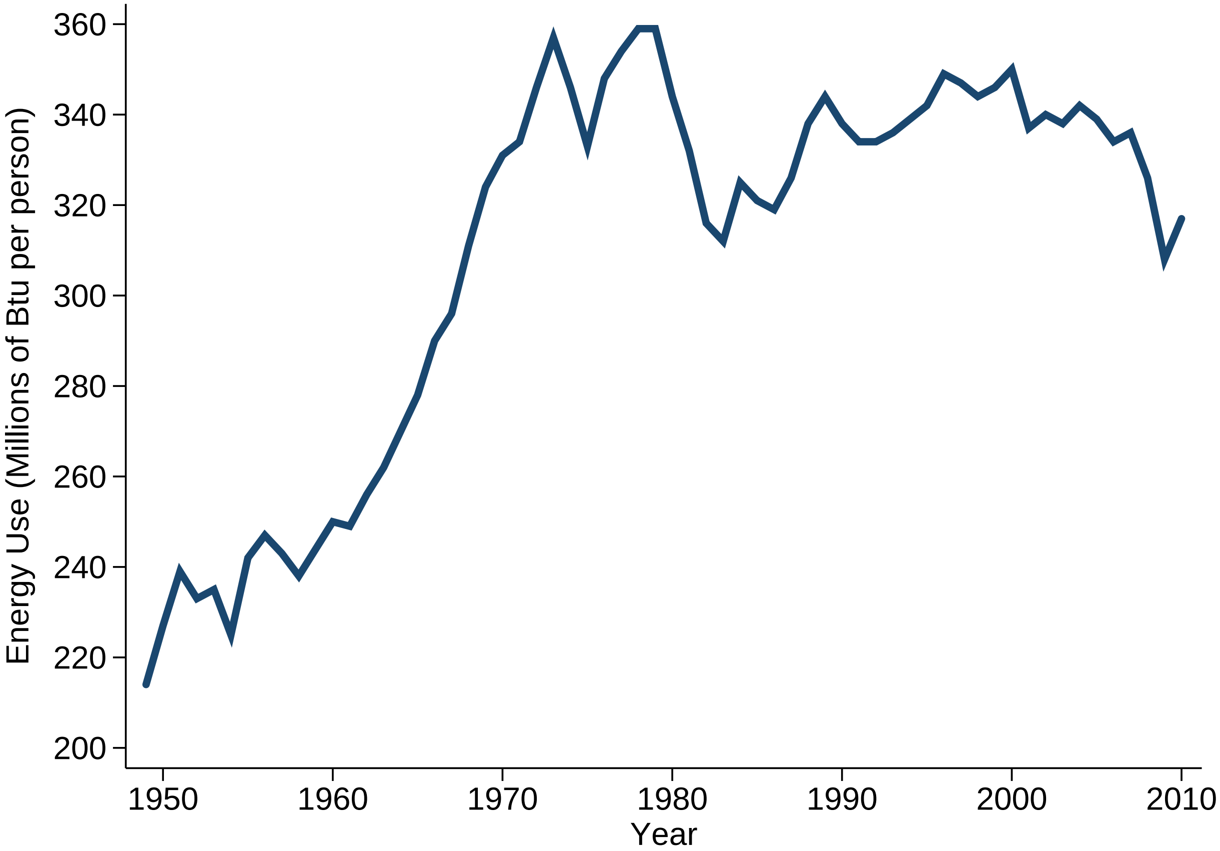
Fossil Fuel's Factor Share v_E

No upward trend - maybe constant?



Energy Use per Person, E/L

No tendency for E/L to fall over time, actually rises



Factor Shares Changing

We don't see massive price increases in resources, as E/L is not falling, and v_E seems to decline somewhat.

Want to be a little more nuanced about factor shares. Let

$$Y = (K^\rho + (BE)^\rho)^{1/\rho} \tag{28}$$

where we ignore labor for now to keep things simple.

This is *constant elasticity of substitution* function. $1/(1-\rho)$ is how EOS between capital and energy. B is energy productivity.

- If $0 < \rho < 1$, then $1/(1-\rho) > 1$ and substitute capital for energy easily (or v.v.)
- If $\rho < 1$, then $1/(1-\rho) < 1$ and cannot substitute capital for energy easily (or v.v.)

Energy's share of output with this function is

$$v_E = \left(\frac{BE}{Y}\right)^\rho \tag{29}$$

Energy's Share

Using

$$v_E = \left(\frac{BE}{Y}\right)^{\rho} \tag{30}$$

what do we expect to happen to energy's share of output? We have, and expect that E/Y is declining over time - value in 2010 about half of that in 1950 (Annual Energy Report, U.S. Energy Department).

Two ways for v_E to decline given E/Y is declining

- If $\rho > 0$, then as E/Y falls, we substitute towards K, and energy isn't used as much. Energy isn't necessary for production.
- If $\rho < 0$, then as E/Y falls, we cannot subsitute towards K, energy is vital to production.

Seems likely that $\rho < 0$, we need energy to produce. So only option left is that B must be rising very quickly to drive down v_E (or keep it from rising).

We appear to be able to innovate away from using resources (e.g. fiber optics for copper) fast enough to avoid rapidly rising resource prices. In the future: ????????

Growth and the Environment

Have not considered resources as anything other than inputs to production at this point. What if we value environmental quality, and resource extraction makes that worse?

Let utility be

$$V = u(C_t) + \rho v(R_{t+1}) \tag{31}$$

where C_t is how much we consume, and R_{t+1} is the stock of remaining resources.

- $u(C_t)$ is utility from consumption, and we assume that it has diminishing marginal utility, or $u'(C_t)$ falls as we consume more
- $v(R_{t+1})$ is utility from the stock of resources left (e.g. untapped Artic wildlife preserves), and it also has diminishing marginal utility. Means that as the stock of resources *falls*, marginal utility gets *higher*.

Question will be how to pick C and R_{t+1} to maximize utility.

Resources and Consumption

Production is

$$C_t = BE_t^{\gamma} L_Y^{1-\gamma} \tag{32}$$

and

$$R_{t+1} = R_t - E_t \tag{33}$$

So using E_t to increase consumption means we have a smaller stock of R_{t+1} left over to enjoy. There's a trade-off in using resources.

Using discrete time, R_{t+1} for simplicity.

Optimization

Optimal use of resources. Take derivative of utility with respect to E_t,

$$u'(C_t)\frac{\partial C_t}{\partial E_t} + \rho v'(R_{t+1})\frac{\partial R_{t+1}}{\partial E_t} = 0 \tag{34}$$

Evaluate the two partial derivatives

$$\frac{\partial C_t}{\partial E_t} = \gamma B E_t^{\gamma-1} L_Y^{1-\gamma} = \gamma \frac{Y_t}{E_t} \tag{35}$$

which comes from the production function, and

$$\frac{\partial R_{t+1}}{\partial E_t} = -1 \tag{36}$$

which comes from the resource equation.

Put those partials into the FOC at the top to get

$$\frac{E_t}{Y_t} = \frac{\gamma}{\rho}\frac{u'(C_t)}{v'(R_{t+1})} \tag{37}$$

Environment and Growth

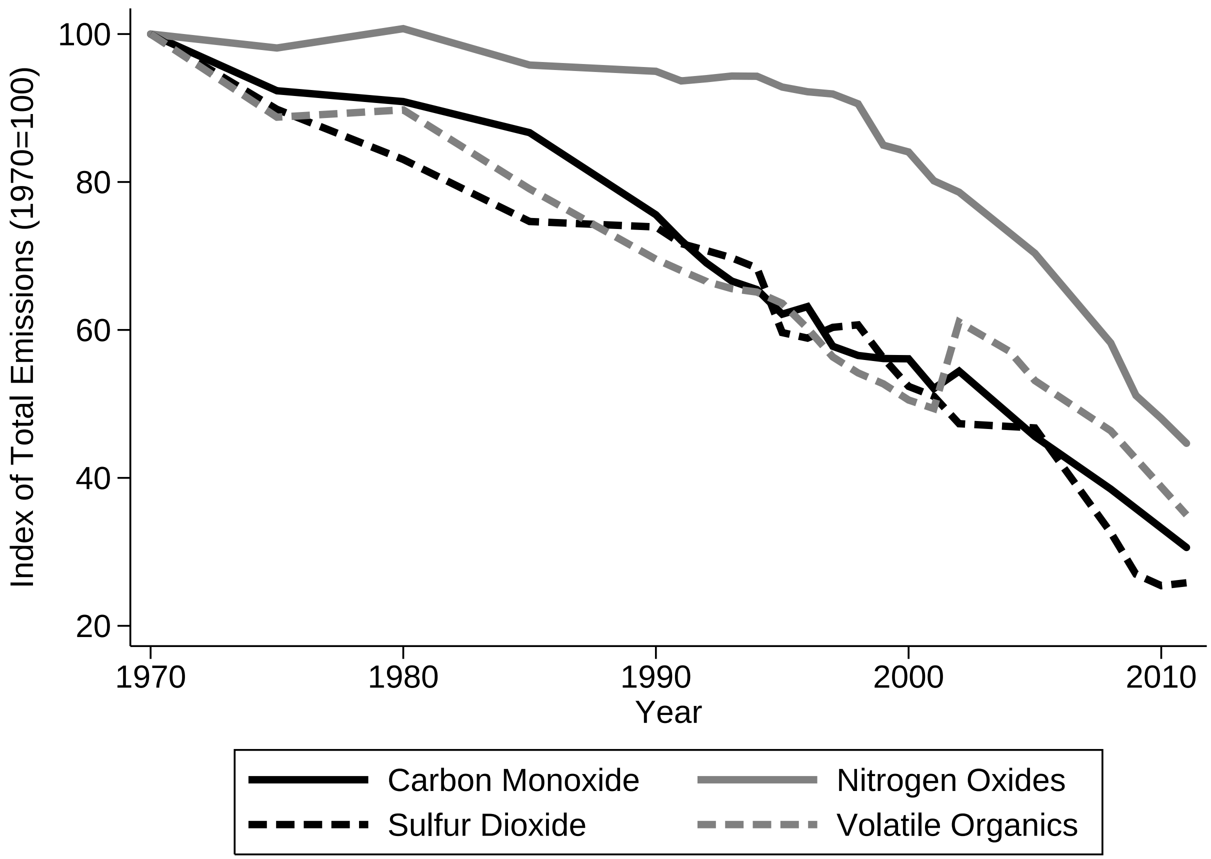
Given the relationship

$$\frac{E_t}{Y_t} = \frac{\gamma}{\rho}\frac{u'(C_t)}{v'(R_{t+1})} \tag{38}$$

- As consumption, C_t, gets high, the marginal utility of consumption, $u'(C_t)$, falls.
- As we use up resources, R_{t+1} gets small, and the marginal utility, $v'(R_{t+1})$, gets big.
- Both these forces suggest that the ratio of E_t/Y_t should *fall* as countries get richer
- For very poor countries, the ratio of E_t/Y_t may *rise* as they get richer, because the marginal utility of resources may be negligible and the MU of consumption remains high

Pollution and Growth

Implies falling pollution (better environment) as we get richer



Factors in Changing Environmentalism

Consider several ways of thinking about environment/growth link

- L_Y are production workers. L_Y may fall if we allocate more people to solving environmental issues, or cleaning up environment, or simply idling them if we want to avoid pollution
- γ is importance of energy in production. One aspect of innovation is lowering γ, which would naturally reduce environmental impact (e.g. solar)
- ρ is our concern for the environment in the future. As we live longer, we may put more weight on utility from R_{t+1} compared to consumption today, lowering our willingness to use energy/resources.

Is there a "right" value for ρ? Nothing this problem can say about that. People speculate/argue over it. What is right weight to put on future generations having access to R_{t+1}?