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10 Some Useful Models

The range and complexity of the models developed in the previous chapters are testimony to the creativeness of macroeonomic theorists. But what good are those models for the working economist? Are they merely "mental gymnastics of a peculiarly depraved type" (Samuelson 1947), or are they useful and are they used?

In fact many of the models we have presented are used not only to clarify conceptual issues but also to explain current events and to help in the design and assessment of macroeconomic policy. In analyzing real world issues, almost all economists are eclectic, drawing on different models for different purposes. Sometimes the models we have analyzed are used as we presented them; more often the basic model has to be developed in a particular direction for the question at hand. Often the economist will use a simple *ad hoc* model, where an ad hoc model is one that emphasizes one aspect of reality and ignores others, in order to fit the purpose for which it is being used.[1]

Although it is widely adopted and almost as widely espoused, the eclectic position is not logically comfortable. It would clearly be better for economists to have an all-purpose model, derived explicitly from microfoundations and embodying all relevant imperfections, to analyze all issues in macroeconomics (or perhaps all issues in economics). We are not quite there yet. And if we ever were, we would in all likelihood have little understanding of the mechanisms at work behind the results of simulations. Thus we have no choice but to be eclectic.[2]

How does a good economist know what model to use for the question at hand? By being a good economist. Much of the art of economics lies in being able to know which unrealistic assumptions are merely peripheral to the issue at hand and which are crucial.[3]

In this chapter we present a (nonexhaustive) list of models that are useful for analyzing real world issues, and in each case we describe or present an application of the model. In the process we review many of the models we have seen, but we also introduce the workhorses of applied macroeconomics such as the IS-LM and the Mundell-Fleming models.

10.1 Equilibrium Models and Asset Pricing

If one believes that imperfections play a crucial role in macroeconomic fluctuations, why would one ever want to use the models developed in chapters 2 and 3, which assume perfectly competitive markets, or their counterparts in chapter 7, which extend the analysis to allow for the presence of uncertainty?

We can think of a number of cases in which this may be appropriate. The Ramsey model can, for example, be used in its normative interpretation. We may ask how an economy should react to an adverse technological shock, an increase in the price of its imports, or an increase in the interest rate it has to pay on its foreign debt. Should it cut consumption or investment, or should it, instead, increase its indebtedness? We have seen in chapter 2 how the open economy version of the Ramsey model can be used to analyze these issues. For issues having to do with intertemporal transfers, such as the burden of the debt and Ricardian Equivalence and the effects of pay-as-you-go social security, the two natural starting points are the Ramsey and the Diamond model with and without bequests. For these issues, imperfections in goods and labor markets may not be central, and it is a good strategy at first to leave them out.[4]

A third application is to asset pricing. Again, imperfections in goods and labor markets may not be crucial to the understanding of the relation between short- and long-term interest rates, or between returns on stocks and returns on bonds. Equilibrium business cycle models that incorporate uncertainty and asset choice by consumers provide a natural environment in which to study these issues. This is what we now do.

Let us return to the maximization problem of a consumer under uncertainty studied in chapter 6. The consumer, who has an horizon of T periods, maximizes

$$E\left[\sum_{t=0}^{T-1} (1+\theta)^{-t}U(c_t)|0\right]. \tag{1}$$

We do not need to specify fully the dynamic budget constraint at this point. All we need to assume is that at time t the consumer can choose to carry his wealth in any of n risky assets, with a (net) stochastic rate of return z_{it}, $i = 1, \ldots, n$, and in a riskless asset, with a rate of return r_t. This implies a set of $n+1$ first-order conditions at time t of the form

$$U'(c_t) = (1+\theta)^{-1}E[U'(c_{t+1})(1+z_{it})|t], \qquad i = 1, \ldots, n, \tag{2}$$

$$U'(c_t) = (1+\theta)^{-1}(1+r_t)E[U'(c_{t+1})|t]. \tag{3}$$

We have shown in chapter 6 how to derive these equations. Their interpretation is straightforward. The consumer must choose consumption such that, along the optimal path, marginal utility this period is equal to discounted expected marginal utility next period. This must be true at the margin no matter what asset, risky or riskless, is considered; this gives us the $n+1$ first-order conditions. For risky assets, what matters is the expected value of the product of the marginal utility and the rate of return; both are uncertain as of time t. For the riskless asset, the rate of return, which is known at time t, can be taken out of the expectation. This gives equation (3).

The Consumption CAPM

Equations (2) and (3) give a set of joint restrictions on the processes for consumption and asset returns. In chapter 6 we thought of equations (2) and (3) as imposing restrictions on the behavior of consumption, given the process for asset returns. But we can, instead, think of them as telling us what the equilibrium asset returns must be, given the process for consumption.

By substituting (3) into (2), we obtain

$$0 = E[U'(c_{t+1})(z_{it} - r_t)|t], \qquad i = 1, \ldots, n. \tag{4}$$

Equivalently, and dropping the time index to denote conditional first and second moments (so that $E[\cdot|t]$ is denoted $E[\cdot]$, etc.),

$$0 = E[U'(c_{t+1})]E[z_{it} - r_t] + \text{cov}[U'(c_{t+1})z_{it}], \qquad i = 1, \ldots, n. \tag{5}$$

Thus the expected return on asset i in an equilibrium satisfies

$$E[z_{it}] = r_t - \frac{\text{cov}(U'(c_{t+1})z_{it})}{E[U'(c_{t+1})]}, \qquad i = 1, \ldots, n. \tag{6}$$

The higher the covariance of an asset's returns with the marginal utility of consumption, the lower is the equilibrium expected return on the asset. With diminishing marginal utility, the implication is that in equilibrium consumers are willing to accept a lower expected return on an asset that provides a hedge against low consumption by paying off more in states when consumption is low.

This equation must hold for any consumer who can freely choose between the $n + 1$ assets. If all consumers are identical and infinitely lived, then equation (6) holds, using the consumption of the representative individual, and, given a specification of utility, can be tested with data on aggregate consumption and asset returns.[5] If individuals are finitely lived but otherwise identical, we have to take into account that the consumption of those who are alive at both times t and $t + 1$ is not exactly the same as aggregate consumption at time t and $t + 1$; if we look at short intervals, however, this is unlikely to be a major issue. If individuals differ in other ways, in the form of the utility function or the presence of nondiversifiable income risk, for example, the conditions under which equation (6) can be tested using aggregate data are not likely to be satisfied (see Grossman and Shiller 1982). Nevertheless, equation (6) gives us a simple way of thinking about the determinants of asset returns.

The CAPM and Betas

Suppose that there exists an asset or a composite asset, m, whose return is perfectly negatively correlated with $U'(c_{t+1})$ [i.e., assume that $U'(c_{t+1}) = -\gamma z_{mt}$, for some γ]. It follows that for all risky assets,

$$\text{cov}[U'(c_{t+1})z_{it}] = -\gamma\ \text{cov}(z_{mt}z_{it}). \tag{7}$$

Further, for asset m, equation (6) implies that

$$E[z_{mt}] = r_t - \frac{\text{cov}[U'(c_{t+1})z_{mt}]}{E[U'(c_{t+1})]}$$
$$= r_t + \frac{\gamma\ \text{var}(z_{mt})}{E[U'(c_{t+1})]}. \tag{8}$$

By substituting (7) and (8) into (6), we obtain

$$E[z_{it}] - r_t = \left[\frac{\text{cov}(z_{it}z_{mt})}{\text{var}(z_{mt})}\right](E[z_{mt}] - r_t), \tag{9}$$

or by defining β_i as $[\text{cov}(z_{it}z_{mt})/\text{var}(z_{mt})]$,

$$E[z_{it}] - r_t = \beta_i(E[z_{mt}] - r_t). \tag{10}$$

This equation is known in finance as the "security market line." If we interpret asset m as the asset composed of all existing tradable assets ("the market"), then equation (10) tells us that the expected return on a given asset in excess of the safe rate is proportional to the expected return on the market in excess of the safe rate. The coefficient of proportionality is equal to the coefficient β_i, which has the interpretation of a regression coefficient of z_{it} on z_{mt}.

This equation implies that a stock with a large variance of returns may or may not require a risk premium for consumers to hold it. If these variations are positively correlated with the market, the asset will have a large β and will indeed require a positive premium. If, however, the variations are uncorrelated with those of the market, the risk in holding the asset can in effect be diversified away, and the equilibrium return will be equal to the riskless rate. If a stock covaries negatively with the market, it provides a hedge, and consumers will be happy to hold it at an expected rate of return that is lower than the riskless rate.

A comparison between equations (6) and (10) shows the attractiveness of (10) for empirical purposes. The equilibrium relation in (10) between the market expected rate of return, the riskless rate, and the rate of return on any asset does not involve the specification of preferences and risk aversion. Computing the beta of a security can be done using a simple regression. As a result betas have become a standard product in applied finance. They are commercially available and are used in stock market analysis. The asset m used in the calculation of the betas is typically the entire stock market.

Although we have derived the security market line from the consumption CAPM (capital asset-pricing model) and the assumption of perfect negative correlation between the return on the market portfolio and the marginal utility of consumption, its derivation actually precedes the derivation of the consumption CAPM. It was derived, in what is now known as the standard or traditional CAPM, by Lintner (1965), Mossin (1966), and Sharpe (1964). The derivation was based on a two-period model in which utility was either defined directly over the mean and variance of portfolio returns or assumed to be quadratic. Merton (1973) showed under what conditions the standard capital asset-pricing formulas could be derived in continuous time from intertemporal optimization of consumers over portfolio choices and consumption.

In essence, the standard CAPM should be a good approximation to asset pricing when the marginal utility of consumption is highly correlated with the return on the stock market, or more generally the portfolio of tradable assets. This is more likely to be the case if most assets are tradable: the presence of a large nontradable asset such as human wealth is likely to decrease the correlation. In this case the consumption CAPM may do better.[6] In practice, however, the consumption CAPM (using data on aggregate consumption) appears to describe asset returns less accurately than the standard CAPM (Mankiw and Shapiro 1986).

The Lucas Asset-Pricing Model

In chapter 6 we thought of the first-order conditions of the consumer optimization problem as imposing restrictions on consumption given asset returns. We have used them above, instead, to think about restrictions on asset returns given consumption. In fact, though there is nothing wrong with either interpretation, both consumption and asset returns are endogenous and respond to shocks affecting the economy. Ultimately, it is these reactions to exogenous shocks that we would like to understand. This is generally very hard. Lucas (1978) cut through the difficulty by considering an exchange economy in which output each period was exogenous and perishable. This in effect makes consumption equal to output in equilibrium and thus exogenous; the first-order conditions can then be used to price assets as a function of (exogenous) consumption. Although this would appear to eliminate the difficulty rather than to solve it, the model has proved extremely useful to study a range of empirical issues. We briefly present it and discuss a few implications.

There are n risky assets in the economy, each of which generates a stochastic physical return in the form of perishable manna, equal to d_{it} per period. (The assets can usefully be thought of as trees, and output as seedless apples.) The assets are the only source of income in the economy. Denote by p_{it} the ex-dividend price of asset i in period t. Let p_t and d_t be the $n \times 1$ vectors of prices and dividends at time t.

The economy consists of identical infinitely lived consumers. The representative consumer maximizes

$$E\left[\sum_{t=0}^{\infty} (1+\theta)^{-t} U(c_t) | 0\right].$$

In any period the consumer receives dividends on the assets that he holds. He then decides how much to consume and what assets to hold into the next period. Let x_{it} be the quantity of asset i that the consumer holds between t and $t+1$. Let x_t be the $n \times 1$ vector of x_{it}. The budget constraint can then be written as

$$c_t + p_t' x_t = (p_t + d_t)' x_{t-1}.$$

The right-hand side gives the value of the portfolio chosen at time $t-1$, as of time t, including dividends. The left-hand side is equal to consumption plus the value of the portfolio chosen at time t. The first-order conditions are

$$p_{it} U'(c_t) = (1+\theta)^{-1} E[U'(c_{t+1})(p_{it+1} + d_{it+1}) | t], \qquad i = 1, \ldots, n. \tag{11}$$

For market equilibrium, the quantities of each asset demanded must be equal to the exogenous supply. Assuming that there is one unit of each asset, equilibrium implies that $x_{it} = 1$ for all i, t. From the budget constraint this implies that $c_t = \sum d_{it}$: consumption must be equal to output, which is the sum of dividends. Thus equation (11) gives us a recursive relation determining the price of the asset as a function of exogenous variables, the d_{it}'s. We can solve equation (11) forward to get, assuming no bubbles,

$$p_{it} = E\left[\sum_{j=1}^{\infty} (1+\theta)^{-j} \left(\frac{U'(c_{t+j})}{U'(c_t)}\right) d_{it+j} | t\right]. \tag{12}$$

The price is equal to the expected present discounted value of dividends, where the discount rate used for $t+j$ is the marginal rate of substitution between consumption at time $t+j$ and consumption at time t. Although the right-hand side only depends on the joint distribution of endowments and is exogenous, it is far from easy to see how it behaves over time. To go further, it is necessary to make assumptions about either the utility function or the distribution of returns.

Assume first that consumers are risk neutral so that $U'(c)$ is constant. Then the price is given by

$$p_{it} = E\left[\sum_{j=1}^{\infty} (1+\theta)^{-j} d_{it+j} | t\right]. \tag{13}$$

This is a familiar formula from chapter 5. The price is equal to the present discounted value of expected dividends, discounted at a constant rate, which is the subjective discount rate of consumers. Movements in prices come from movements in expected dividends. It is this pricing formula that is often tested in the volatility tests described in chapter 5.

Assume, instead, that consumers are risk averse. Assume that there is only one asset (or equivalently that we are looking at the price of the market portfolio), with dividend d_t, so that $c_t = d_t$. Assume further that utility is logarithmic. Then the price p_t of the single asset is given by

$$p_t = E\left[\sum_{j=1}^{\infty} (1+\theta)^{-j} \left(\frac{d_t}{d_{t+j}}\right) d_{t+j} | t\right] = \left(\frac{1}{\theta}\right) d_t.$$

In this case the price of the stock depends only on the current dividend, and not on future expected dividends. This is in sharp contrast with the previous case. Two things happen when consumers expect higher dividends at time $t + j$. The first is, as before, that at given marginal rates of substitution, higher dividends increase the price. But higher dividends also mean higher consumption, and thus lower marginal utility: other things being equal, dividends are valued less when times are good and consumption is high. Put another way, higher dividends in this case are associated with increases in interest rates. The result, in the logarithmic utility case, is to leave prices unchanged. This example is a useful, if somewhat extreme, counterexample to the idea that higher expected dividends necessarily increase prices.

Why is the Lucas model of more than academic interest? Because equation (12) holds whether or not the economy is an exchange economy. If the representative agent assumption is correct or, more generally, if aggregation conditions are satisfied, equation (12) holds, given the process for aggregate consumption, so long as agents can freely choose the composition of their portfolio. Thus substantial research has gone into asking whether, given the actual process followed by aggregate consumption, the pricing of assets is roughly consistent with equation (12). Campbell (1986), for example, has used this model to study how the process for consumption determines the term structure of interest rates, that is, the relation between yields to maturity on bonds of different duration. Mehra and Prescott (1985) have examined whether this model can explain the large premium of average stock returns over riskless bonds, which for the United States has been historically around 6%.[7] They parameterize the consumption process as a Markov process for the growth rate of consumption so as to fit the postwar process for consumption. They then derive the risk premium and the riskless rate as a function of alternative values of the discount rate and the degree of relative risk aversion. Their conclusion is that there is no set of parameters that can explain both the riskless rate and the the equity premium. The size of the equity premium is much larger than can be obtained from any plausible estimate of risk aversion. This result has triggered further research on how heterogeneity in consumers may explain the empirical premium (e.g., Mankiw 1986). This research has potentially important implications, for example, on how to think about the golden rule under uncertainty.

10.2 Money Demand Models, Deficits, Seigniorage, and Inflation

We used money demand models in chapter 4 to study the dynamics of inflation. We argued there that the use of such ad hoc models, which implicitly assume that real variables are either constant or move slowly

Table 10.1
Deficit/GNP ratio and inflation in Israel, 1974–1986

	1974	1975	1976	1977	1978	1979	1980
Deficit/GNP (%)	18.8	23.5	12.1	14.9	18.2	11.1	11.6
Inflation (%)	56	23	38	43	48	111	133
	1981	1982	1983	1984	1985	1986	
Deficit/GNP	20.6	11.3	4.9	16.9	4.2	−7.1	
Inflation	101	131	191	445	185	20	

Note: Inflation = rate of change of the CPI.

enough compared to nominal variables that their movement can be ignored, could be appropriate when the focus is on times of high inflation. We now extend our earlier analysis of the relation between deficits and inflation.

Students of hyperinflation (e.g., Sargent 1982; Dornbusch and Fischer 1986) often stress the role of the budget deficit in the inflationary process, reporting that fiscal reform seems, in practice, to be an essential component of a stabilization program. A common criticism of this stress on the budget deficit is that the data rarely show a strong positive association between the size of the budget deficit and the inflation rate. Table 10.1 illustrates this fact with annual data on the budget deficit as a percentage of GNP and the inflation rate in Israel over the period 1974 to 1986.

One possible explanation is that deficits are in fact associated, at different times, with different expectations of money growth in the future. In the face of high deficits, people may anticipate that the government will have no choice but to increase its use of seigniorage. Or they may expect that under such pressure the government will be forced to introduce drastic fiscal reform and to reestablish budget balance through higher taxes. These are likely to lead to different inflation rates today. This argument is explored by Drazen and Helpman (1988) and has the following structure:[8]

Consider an economy in which money demand is given by

$$\frac{M}{PY} = \phi(r + \pi^*), \qquad (14)$$

where Y is output, r is the real rate of interest, and π^* is the expected rate of inflation; both Y and r are assumed to be constant in what follows. Money demand is decreasing in the nominal interest rate: $\phi'(\ ) < 0$. Furthermore it is assumed that $\pi\phi(r + \pi)$ is increasing in π for $\pi < \pi'$ and decreasing in π for $\pi > \pi'$. This implies that the graph of seigniorage revenue against the inflation rate has the Laffer curve property.