Final Exam Answers ECGA7020 **D. McLeod Fall 2005**

1. Simple RBC model (16 points)
2. Precautionary savings. (16 points)
3. **Choosing a Central Banker (16 points)**
4. Portfolio choice (12 points)
5. Asset pricing (12 points)
6. *Ramsey model with taxes (6 points)*

3. Choosing the right Central Banker: In the Kydland-Prescott/Barro-Gordon framework the Central bank faces a large number of private agents seeking to predict inflation accurately that is to minimize, $MinE_{t-1}(\pi_t-\pi_t^e)^2$. Luckily, the Central Bank (CB) sets π_t to minimize its loss function $L=\min\left\{(y_t-y^*)^2+a(\pi_t)^2\right\}$ given a Lucas supply curve: $y_t=\bar{y}+b(\pi_t-\pi_t^e)-z_t$ after observing a supply shock z_t and the private sector's best inflation forecast, π^e. The CB's targets for output, y*, is a bit ambitious in that. $y^*>\bar{y}$. Alternatively, think of a distortionary wedge $k=y^*-\bar{y}$ between desired and average output resulting from monopolistic market power or taxes, denoting this gap as k as in Obstfeld & Rogoff section 9.5.

(a) Solve for an equilibrium inflation rate π_t and π_t^e keeping in mind that the $z_t \sim N(0,\sigma^2)$ and simplifying by setting b = 1. Substituting the supply function into the loss function given above,

$L=\min\left\{(\bar{y}+b(\pi_t-\pi_t^e)-z_t-y^*)^2+a(\pi_t)^2\right\}$ the CB minimizes its loss function by setting,

$$\frac{\partial L}{\partial \pi_t}=0\Rightarrow 2(\bar{y}+b(\pi_t-\pi_t^e)-z_t-y^*)+2a(\pi_t)=0\Rightarrow \pi_t(1+a)=(y^*-\bar{y})+\pi_t^e+z_t)\Rightarrow$$

$$\pi_t=\frac{1}{(1+a)}(k+\pi_t^e+z_t)\quad \textit{setting } k=y^*-\bar{y} \textit{ and } b=1\ \Rightarrow \pi_t^e=\frac{1}{(1+a)}(k+\pi_t^e)\ \textit{as}\ E(z_t)=0.$$

Rational expectations implies $E_t\pi_t=\pi_t$ so in equilibrium $\pi_t^e=\frac{k}{a}$ substituting back into the expression for π_t yields the actual inflation rate, $\pi_t=\frac{k}{a}+\frac{z_t}{(1+a)}$ or, $\pi_t=\pi_t^e+\frac{z_t}{(1+a)}$

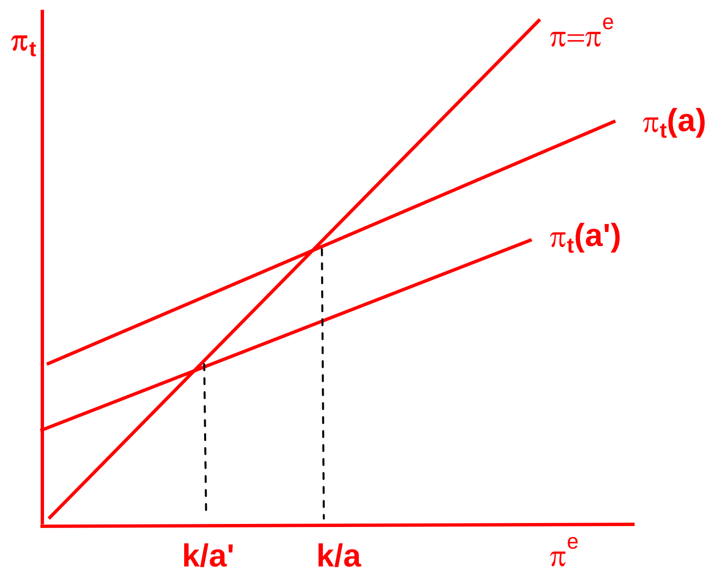
Show the equilibrium y_t and the amount of output stabilization that takes place under this regime, given the Lucas supply curve. To determine the amount of stabilization we substitute these expressions for inflation and expected inflation back into the Lucas supply curve and compute the new variance of output,

$$y_t=\bar{y}+b(\pi_t-\pi_t^e)-z_t\ \textit{becomes}\ y_t=\bar{y}+(\frac{k}{a}+\frac{z_t}{1+a}-\frac{k}{a})-z_t\Rightarrow y_t=\bar{y}-\left(\frac{a}{1+a}\right)z_t$$

$$\textit{so the}\ \mathrm{var}(y)=\left(\frac{a}{1+a}\right)^2\mathrm{var}(z_t)=\left(\frac{a}{1+a}\right)^2\sigma^2.$$

Some stabilization takes place, in the sense that the variance of y falls by the fraction $\left(\frac{a}{1+a}\right)^2$ assuming a > 0 (where "a" is the weight the CB loss function puts on inflation).

Use the graph plotting π_t against π_t^e to show how a more conservative central banker a' > a affects both the equilibrium inflation rate and the amount of output stabilization.



Suppose the L(a) and not L(a') is society's loss function (SWF). Is society always better off appointing a conservative central banker? What is the tradeoff society faces (hint: compute the variance of y_t with and w/o a conservative central banker)? A conservative CB a' > a reduces expected and actual inflation but increases the variance of output since $\left(\frac{a'}{1+a'}\right)^2 > \left(\frac{a}{1+a}\right)^2$ -- there is a tradeoff between inflation and the variability of output (stabilization). If society's preferences are a < a' it may more variable output than it desires (because the conservative CB puts too much weight on inflation). However, this cost is partially offset by lower inflation.

b) One way for society to get lower inflation rate is to offer the CB head a bonus ω linked to the realized inflation rate as in these two schemes,

$$L_1 = \min\left\{(y_t - y^*)^2 + a(\pi_t)^2 + 2\omega^*\pi_t\right\} \text{ or } L_2 = \min\left\{(y_t - y^*)^2 + a(\pi_t)^2 + 2\omega z_t \pi_t\right\}$$

Compare the impact of the two contracts: the optimal bonus in the first scheme is ω= k, explain. Using the same solution strategy for π and π^e as in part (a) using L_1, yields, $\pi_t^e = \frac{(k-\omega)}{a}$ and

$\pi_t = \frac{k-\omega}{a} + \frac{z_t}{(1+a)}$ while the variance of y remains unchanged: $\text{var}(y) = \left(\frac{a}{1+a}\right)^2 \sigma^2$. Note that the bonus at $\omega = k$ reduces expected inflation to 0 and actual inflation to $z_t/(1-a)$ leaving the amount of stabilization or var(y) unchanged. Using the contract scheme L_2 allows the government to influence the amount of output stabilization (but not expected inflation) as one obtains,

$$\pi_t^e = \frac{k}{a} \text{ and } \pi_t = \frac{k}{a} + \frac{(1-\omega)}{(1+a)} z_t \text{ while } y_t = \bar{y} - \left(\frac{a+\omega}{1+a}\right) z_t \text{ and } \mathrm{var}(y) = \left(\frac{a+\omega}{1+a}\right)^2 \sigma^2$$

What is the advantage of the second scheme where the bonus is linked to the supply shock z_t particularly when one is dealing with a conservative central banker? The advantage of an L_2 contract linking the bonus ω to the supply shock z_t is that the public can influence the extent to which the central bank engages stabilizes y. When $\omega < 0$ the variance of y falls but the variance of π increases. One disadvantage of L_2 contracts is that expected inflation remains unchanged. However, with a conservative central banker a' > a expected inflation k/a' is low anyway. Since the variability of y rises with "a" contract that lowers var(y) but leaves expected inflation unchanged may be more desirable.

Rogoff (2004) argues globalization has made the economy more competitive, reducing k. How does a reduction in k affect long term inflation and stabilization (the variance of y_t). In the absence of a perfect L_1 contract (i.e. $\omega = k$) globalization reduces k and expected inflation $\pi^e = k/a$ in all cases described above, while having no effect on the amount of stabilization or var(y) since only "a" and ω with L_2 style contracts, affects var(y).

c) Suppose a new CB head introduces an inflation target $\pi^* > 0$ modifying the CB loss function above to, $L = \min\left\{(y_t - y^*)^2 + a(\pi_t - \pi^*)^2\right\}$ compute the new π_t and π_t^e.

What are the consequences of raising π^* for output stabilization and expected inflation? Is there any benefit from raising target inflation from 0 to π^*? Suppose proposed Fed Chair Bernanke's was an anti-deflation hawk: why would he be happier with $\pi^* > 0$ as his inflation target?

With an inflation target,

$$\pi_t^e = \frac{k}{a} + \pi^* \text{ and } \pi_t = \frac{k}{a} + \frac{z_t}{(1+a)} + \pi^* \text{ while } y_t = \bar{y} - \left(\frac{a}{1+a}\right) z_t \text{ and } \mathrm{var}(y) = \left(\frac{a}{1+a}\right)^2 \sigma^2$$

Inflation targeting raises actual and expected inflation as long as $\pi^* > 0$ (the previous implicit target was $\pi^* = 0$), leaving var(y) unchanged. However one advantage of inflation targeting is that as long as $\pi^* > 0$ the probability of deflation decreases (that is the prob$\{\pi_t < 0 \mid z_t\}$ falls). A "deflation hawk" such as incoming Fed Chair Ben Bernanke may wish to avoid deflation at all costs so a higher target is better than zero. Inflation targeting with $\pi^* > 0$ provides a deflation hawk more scope for output stabilization in the face of large negative realizations of z_t (booms?).