ECGA 7020 Macroeconomics II Midterm Answers Fall 2005 D. McLeod

Midterm Question I-2. Different Ramsey Peoples: Baker-DeLong and Krugman (2005) or BDK argue it makes sense to incorporate population or family preferences directly into the utility function of a Ramsey style model. Preferences for future consumption, for example, may depend on how many children one has or, according to BDK (2005), how many new immigrants enter the country. Using the present value Hamiltonian (here is a current value H solution):

$$H = e^{-(\rho n^{\varepsilon})}\{u(c_t) - \lambda_t[f(k) - c - \delta k]\}$$

where δ is the depreciation rate and $u(c_t) = \frac{c^{1-\theta}}{1-\theta}$. The term n^{ε} reflects household head's altruism toward future generations. As ε falls toward zero, agents become increasingly altruistic. When $\varepsilon = 0$ until they are perfectly altruistic and we are back in the standard Ramsey model (but in the above case with depreciation instead of labor augmenting technical progress g and positive population growth).

a) Assuming CRRA utility set up and solve this problem for $\frac{\dot{k}}{k}$ *and* $\frac{\dot{c}}{c}$.

Using the PV Hamiltonian,

$$H = e^{-\hat{\rho}t}[u(c_t) + q_t \cdot g(k,c)] \quad where \ \hat{\rho} = \rho n^{\varepsilon}$$

$$where \ \ g(k,c) = f(k) - c - \delta k$$

Taking the FOC with respect to c and k,

$$H_c = u'(c_t) + q_t g_c \quad where \ g_c = -1 \Rightarrow u'(c_t) = q_t$$

$$-H_k = \frac{d(q_t e^{-\hat{\rho}})}{dt},$$ differentiating w/r to time,

$$-e^{-\hat{\rho}t} q_t(f'(k) - \delta) = e^{-\hat{\rho}t}(\dot{q}_t - \hat{\rho} q_t) \ \Rightarrow -q_t(f'(k) - \delta) = \dot{q}_t - \hat{\rho} q_t$$

dividing both sides by q_t, $-(f'(k) - \delta) + \hat{\rho} = \frac{\dot{q}_t}{q_t}$.

Given $u(c_t) = \frac{c^{1-\theta}}{1-\theta}$ we can solve for the time path of consumption since,

$$c^{-\theta} = q_t, \text{ or in log form, } -\theta \ln c = \ln q_t \ \Rightarrow \ \frac{\dot{c}}{c} = -\frac{1}{\theta}\frac{\dot{q}_t}{q_t}$$

substituting this into the f.o.c. for k,

$$\frac{\dot{c}}{c} = \frac{(f'(k) - \delta) - \hat{\rho}}{\theta} \quad where \ \hat{\rho} = \rho n^{\varepsilon}$$

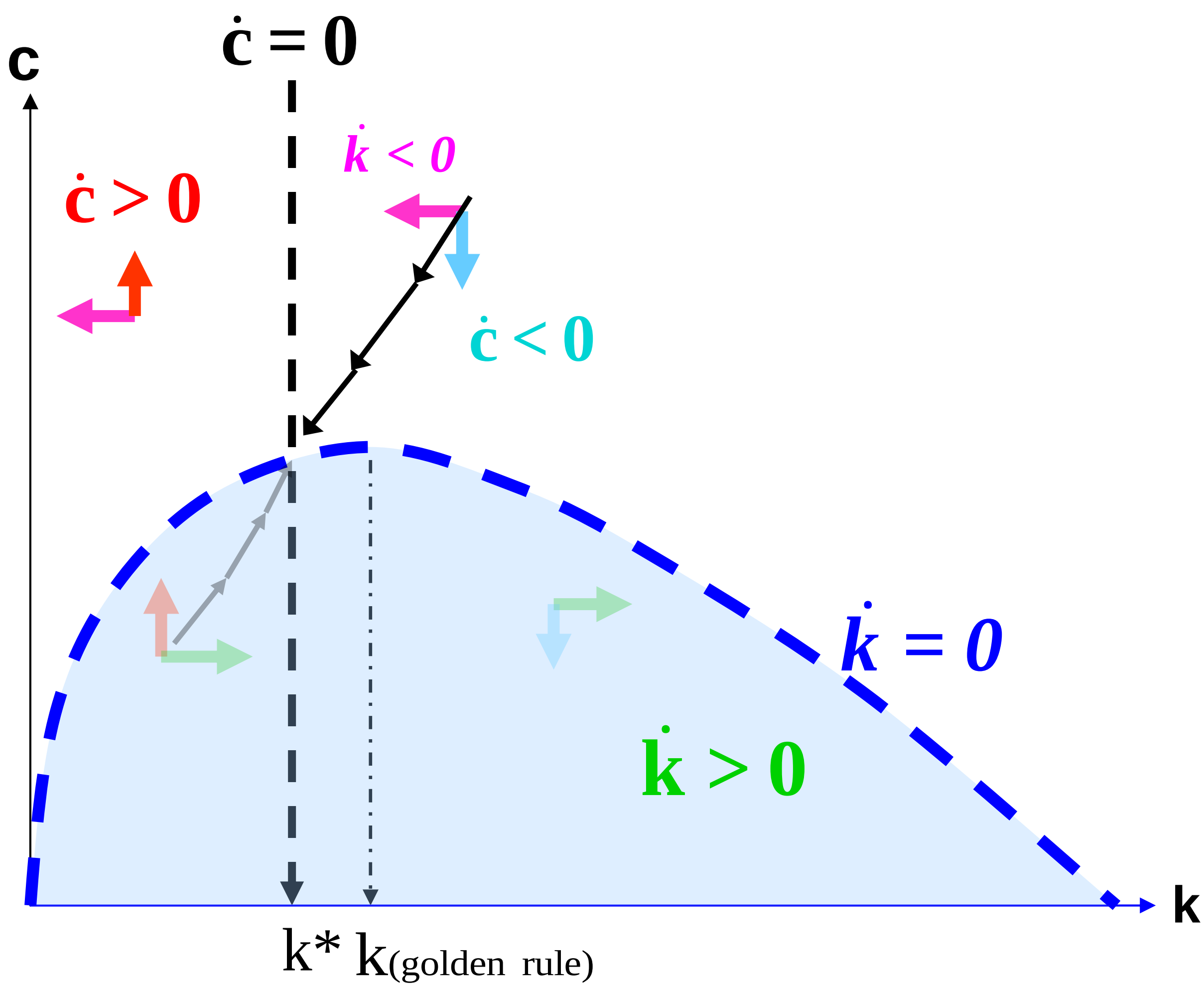
$$Since \ \ \dot{k} = sy - \delta k = f(k) - c - \delta k \Rightarrow \frac{\dot{k}}{k} = \frac{f(k) - c}{k} - \delta$$

b) What is the new "modified golden rule" and standard golden rule for this economy?

The standard golden rule maximizes consumption. Since c = f(k)-δk consumption is maximized

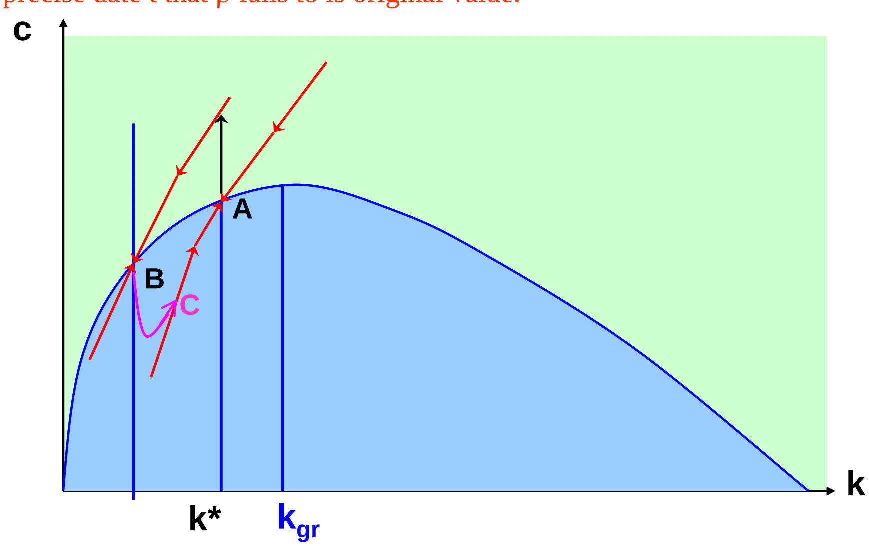
when f '(k) = δ : this is the golden rule. The modified golden rules requires implies that $\frac{\dot{c}}{c} = 0$ *which implies* : $f'(k) = \delta + \hat{\rho}$ *where* $\hat{\rho} = \rho n^{\varepsilon}$, as long as $\hat{\rho} > 0$ saving is painful and consumption never reaches the golden rule maximum and $k^* < k_{gr}$.

c) Use the laws of motion from (a) to solve this model graphically in the usual phase plane diagram with c on the vertical axis and k on the horizontal axis.



d) Assume the government unexpectedly announces a new immigration policy which raises ε. What happens to consumption and the steady state capital stock and the return on capital? According to BDK (2005) a increase in immigration causes the typical infinitely lived consumer to care less about future generations, raising the discount rate $\hat{\rho} = \rho n^{\varepsilon}$. The modified golden rule is restored with a higher MPK or f '(k) so k falls. Why does c jump up or down? Show this in the phase diagram.

An rise in ρ reduces k*, but to get from point A to B, Consumption jumps (savings falls) to the saddle path associated with new steady state c at B. An anticipated fall in ρ causes consumption to fall returning to the A saddle path on the precise date t that ρ falls to is original value.



Do the same for a fall in n given 1 > ε > 0—A fall in n reduces $\hat{\rho} = \rho n^{\varepsilon}$ causing an increase in k* and a fall in the rate of return to capital, f '(k) – the result that BDK (2005) were looking for.

e) Congress revolts and announces the President's immigration plan will terminate in two years. Show the dynamics of c and k during this two year period before the plan ends in T+2.

Returning to the diagram above, consumption falls and rolls to the right so as wind up back on the saddle path for point A the day the immigration law expires and ε returns to its original level as shown by the movement from point B to C in diagram above.

d) Explain why population growth has no effect on the rate of return and k* in the standard Ramsey model, whereas it does in the Solow and OLG model.

The typical Ramsey household lives forever and maximizes consumption per person. Since each person is identical across generations, an increase in population has no effect on k*. In the Solow model the savings rate is exogenous, so an increase in n spreads the same investment over a larger number of people, so k* falls. In the OLG model, each generation lives only two periods, so an increase in population spreads savings of the young over a larger pool of new workers, hence k* falls. (see also Mankiw's explanation on page 316-17 of BDK(2005)).