General Instructions: Answer three of questions from the two groups of questions outlined below. The emphasis is not on getting the correct answer, but showing how one might derive the correct answer. It is therefore important to set problems up carefully and explicitly in steps (using the left exam book page for algebra). Use the solution approach and strategies emphasized in class (for example, maximize the Bellman equation using terminal wealth as your state variable). You have about 45 minutes per question, max. Start each equation in a separate exam book, clearly labeling the question you are answering using the numbers and short titles below. Put the section you are writing about: 4b,4c, etc. on each page. Start new sections on a new page. Explain key results briefly and clearly, labeling axes and points on figures or drawing arrows to answers. Put figures on the left sheet of the exam book, if that helps. Relax and enjoy the exam.

Choose 2 questions from group I:
1. A Canonical OLG model
2. Different Ramsey Peoples
3. A DP Current Account Policy Function
4. Benevolent-Barro Government?

Choose (1) One question from this group:
5. Coping with Savings gluts in the MFD model
6. A Hybrid model of Perpetual youth.
7. Phase diagram for q theory investment tax.

Question 1. A Canonical OLG model and Dynamic Efficiency

Suppose overlapping generations maximize, $U_t = \ln c_{1t} + \beta \ln c_{2t+1}$

Making use the production function: $F(K_t, L_t) = AK_t^{\alpha} L_t^{1-\alpha}$.

(a) Set up the Langrangian for this problem (suggestion: use on constraint for each period)

(b) Solve for the workers savings rate recalling that $k_{t+1} = s_t w_t = w_t - c_t$

(c) Solve for the standard golden rule savings rate, give that c = y – I which in this model is $c = Ak^{\alpha} - k$ where k = K/L.

(d) Compare the OLG savings rate to the golden rule savings rate (with wages and savings all computed at k_{gr}. What conditions on preferences and technology make dynamic inefficiency more likely in this model?

Question I-2. Different Ramsey Peoples: Baker-DeLong and Krugman (2004) argue it makes sense to incorporate population directly into the utility function of a Ramsey style model. Preferences for future consumption, for example, may depend on how many children one has or on how many immigrants are coming into the country. Written out as a present value Hamiltonian:

$$H = e^{-(\rho n^{\varepsilon})}\{u(c_t) - \lambda_t[f(k) - c - \delta k]\}$$

where δ is depreciation and $u(c_t) = \frac{c^{1-\theta}}{1-\theta}$ and n^{ε} reflects parents altruism towards their children. As ε falls toward zero, parents become increasingly altruistic until they are perfectly altruistic and we are

back in the standard Ramsey model as present by Romer (but with depreciation instead of labor augmenting technical progress which is simpler).

a) Assuming CRRA utility set up and solve this problem for $\frac{\dot{k}}{k}$ *and* $\frac{\dot{c}}{c}$.

b) What is the new "modified golden rule" and standard golden rule for this economy?

c) Use the laws of motion from (a) to solve this model graphically in the usual phase plane diagram with c on the vertical axis and k on the horizontal axis.

d) Assume the government unexpectedly announces a new immigration policy which raises ε. What happens to consumption and the steady state capital stock and the return on capital? Why does c jump up or down? Show this in the phase diagram. Do the same for a fall in n given 1 > ε > 0.

e) Congress revolts and announces the President's immigration plan will terminate in two years. Show the dynamics of c and k during this two year period before the plan ends in T+2.

d) Explain why population growth have no effect on the rate of return and k* in the standard Ramsey model, whereas it does in the Solow and OLG model

Question I-3. Dynamic Programming CA Policy Functions: Assuming log utility, $u(c_t) = \ln c_t$

$$Max \sum_{t=0}^{\infty} \beta u(c_t) \qquad (0.1)$$

$W_{t+1} = (1+r)(W_t - c_t)$ assuming r_t is the fixed world interest rate and wealth at date t is,

$W_t = (1+r)B_t + \tilde{Y}_t$ *where* $\tilde{Y}_t = \sum_{S=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} Y_s$ *is the present value of future* (exogenous) output at date t.

(a) Uses the form of the Bellman equation shown below and solve this problem using W_{t+1} as the control variable:

$$V(W_t) = \max_{c\in(0,W)} u(c) + \beta V(W_{t+1}) \qquad \text{or,} \qquad V(W_t) = \max_{W_{t+1}\in(0,W)} u(W_t - W_{t+1}) + \beta V(W_{t+1})$$

Be sure to show each step in the derivation. Recall the basic strategy of Adda and Cooper or Obstfeld and Rogoff, rewriting $u(c_t)$ as a function W_{t+1}. Note the Euler equation for this problem. You can check your answer however, you want, but only write out the forward looking solution using W_{t+1} as your control variable. You want to end up with a stationary policy function of the form: $W_{t+1} = \varphi(W_t)$.

b) Use $W_{t+1} - W_t$ to derive an expression for the current account, $CA_{t+1} = B_{t+1} - B_t$.

c) Discuss how a fall in the interest rate r affects optimal growth of wealth, consumption and the current account. Compare this result with the results obtained in class for CRRA utility:

$$c_t = \left[1 - \beta^{\sigma}(1+r)^{\sigma-1}\right]W_t \qquad \text{and} \qquad W_{t+1} = \left[\beta^{\sigma}(1+r)^{\sigma}\right]W_t$$

where $\sigma = 1/\theta$ in Romer's notation (the IES). Intuitively, why does the fall in r have a different effect depending on the utility function?

(d) Use Bellman's principle of optimality to explain why choosing maximizing with respect to W_{t+1} automatically maximizes today's consumption c_t and all future consumption. How does this way of thinking about optimization generate a stationary policy function (if one exists)? The answer to this question will prove for adding optimizing under uncertainty.

Question 4: A Benevolent Government circa Barro (1990). Individuals own firms and produce output using a production function that depends in part on public infrastructure spending or investment g, (roads, ports, Internet, etc.). Individuals maximize utility $u(c_t)$ in the usual decentralized fashion

$$\max_{c_t} \int_0^{\infty} \frac{c_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt$$ subject to $\dot{k}_t = (1-\tau)Ak_t^{1-\alpha}g^{\alpha} - c_t$. Production function is given by

$y = Ak^{1-\alpha}g^{\alpha}$ where $0 < \alpha < 1$.

Public expenditure g is financed by a proportional tax on income so that $g = \tau y$. The government cannot borrow; hence it must always have a balanced budget. All variables are expressed in per capita terms but note that labor is not required to produce y (assume k includes human capital too).

a) Set up a present value Hamiltonian to solve this model and find the law of motion for c and k (consumption and the capital stock). Derive the decentralized competitive solution.

b) Show that there is no steady-state level of output or capital k* as this is a endogenous growth model. Show that this is model has a linear AK production function where in this case $A^*(\tau)$. .

c) Solve the model from the point of view of a benevolent growth maximizing government (Central planner). Carefully explain and show why the decentralized solution of this model is not socially optimal.

d) *E.C. What is the optimal tax rate τ or g/y for both solutions.*

II-5. Coping with Prof. Bernanke's savings glut a MF-Dornbusch world:

Lower case letters indicate natural logs, for instance $p = Ln(P)$ is the natural log of the domestic price (P), and for Goods Market: GDP or supply of y^s is fixed $y^s = \bar{y}$ (5.1)

Demand for domestic output depends on the real exchange rate via the trade balance (the 1st term) and the impact of the real interest rate on investment and durables consumption (σ):

$$y^d = \delta(s + p^* - p) - \sigma(i - \dot{p}) + g, \qquad (5.2)$$

where s is the natural log of the nominal exchange rate, i is the nominal interest rate, $\dot{p}$ is the actual and expected rate of inflation, and g is the natural log of government spending.

Price adjustment: **i**f demand exceeds supply, prices increase in proportion to excess demand.

$$\dot{p} = \alpha(y^d - y). \qquad (5.3)$$

Asset market: a standard LM schedule.

$$m - p = \phi y - \lambda i, \qquad (5.4)$$

Where m is the natural log of nominal money supply (M), ϕ and λ are positive constants.

Uncovered interest parity: $i = i^* + \dot{s}$ (5.5)

Combining the above equations yields the laws of motion

$$\dot{s} = \frac{\phi y - m + p}{\lambda} - i^* \qquad (5.6)$$

$$\dot{p} = \frac{\alpha}{1 - \alpha\sigma}\{\delta(s + p^*) - (\delta + \frac{\sigma}{\lambda})p - \sigma\frac{\phi y - m}{\lambda} + g - y\} \qquad (5.7)$$

Using a phase diagram with the exchange rate (s) in the y-axis and price (p) in the x-axis answer the following question graphically. The Mundell-Fleming Dornbusch model is very intuitive, be sure to discuss the economic principles behind your results (e.g. the real exchange rate and the trade balance, floating, if not jumping, nominal exchange rates, slow price adjustment, etc.)

a) An increase in government expenditure (g) – what falls to accommodate the increase in g? Does this improve of worsen the CA? (assuming no foreign debt right now).

b) A world-wide savings glut leads to fall in international interest rates: i*. How does this affect the trade balance?

c) The new Central Bank head wants to avoid deflation, so he decides to offset any changes in its price level caused by the fall in i* by using monetary policy (a change in m) to restore the previous level of p. Show the impact of this change in m, graphically, including the trajectory of the jumping variable the nominal exchange rate s. Why not just change s to offset the effect of a fall in interest rates? Recall throughout this exercise the i* remains lower than its initial level.

II-6. A q-theory Investment Tax: (a) Set up and write down the FOC for an investment model with CRS adjustment costs and a profits tax using a current value Hamiltonian. Use the Cobb-Douglas CRS production function to simplify the firm's profit function and introduce a tax on capital (1+τ)rK where r is the return on capital and V_t is the present value of the firms future profits at date t.

$$V_t = \int_0^\infty e^{-rt}\{F(K_t, L_t) - wL_t - I_t - \psi_t\}dt \qquad (6.1)$$

where F(K,L) is some constant returns to scale (e.g. Cobb-Douglas) production function and wL is the cost of labor inputs. But does V_t = qK? That is, does average q equal marginal q (V_t). Given the adjustment cost function,

$$\psi_t(I,K) = \frac{\varphi I_t^2}{2K_t}. \qquad (6.2)$$

The following system of differential equations derives from the q-theory:

$$\frac{dq}{dt} = rq - \frac{(q+1+\tau_C)^2}{2\varphi} - Af'(k) \qquad (6.3)$$

$$\frac{dk}{dt} = (q-1+\tau_C)\frac{k}{\varphi} \qquad (6.4).$$

Where r the interest rate, A is a parameter of permanent supply shock, t_c is tax credit on investment and φ is a parameter of the cost function of investment that is given by $\psi(I,K) = \frac{\varphi I^2}{2K}$.

b) Draw the phase diagram, set up the Jacobian matrix of this SDE, the q and k is steady state and the slope of the saddle path. In your graph show q in the y-axis and k in the x-axis and explain carefully if there are or not shifts in the $\dot{q}$ and $\dot{k}$ curves.

Use this diagram to explain the:

6A) A permanent but unanticipated decrease in the world/national interest rate (r) due to savings gut. Explain what happens to stock prices (q).

6B) A temporary reduction in the tax on profits (t_c) to end at date T+2. Explain the trajectory of stock prices carefully.

6C) A permanent negative supply shock.

E.C. Finish solving the model and indicate how one might prove marginal equals average q in this context. Why is this a useful result? Why is the industry and firm level capital stock identical in this model but not in the Romer π(K). What is advantage of his approach?

II-7 Hybrid Perpetual Youth in continuous time: This OLG model with uncertain lifetimes represents a sort of hybrid OLG model with Ramsey like characteristics. It gets its name from the simplifying assumption that both young and old have roughly the same probability of dying, π in this case (with genetic engineering this may not be that far-fetched). Doing all the hard work for you (see Blanchard and Fischer (1989) chapter 3 for details) we end up with the following laws of motion system of differential equations:

$$\frac{\dot{c}}{c} = [f'(k) - \delta + \alpha - \rho]c - \pi(\pi + \rho)k$$

where ρ is the discount rate, α the exponent of a Cobb-Douglas production function and δ is the depreciation rate. The law of motion for capital looks very Ramsey like:

$$\frac{\dot{k}}{k} = f(k) - \delta k - c\,.$$

a) Using the same c-k space (c is the jumping variable on the vertical axis) as you did for the Ramsey model, solve this system graphically, identifying the golden rule for k, the modified golden rule for c.

b) Does this model exhibit saddle point stability? How can you tell?

c) E.C. Is there a potential for dynamic inefficiency in this model? Outline the conditions under with this might occur.

8. EC. Only if you have extra time: Some Search-Unemployment puzzles: recall that workers and firms negotiate to maximize,

$$\max_w \left\{(E-U)^{\beta}(J-V)^{1-\beta}\right\}$$ which using eq. 1.6 and 1.9 from the handout is,

$$\max_w f\left\{\left[\frac{w-b}{r+s+\theta^{1-\alpha}}\right]^{\beta}\left[\frac{y+c-w}{r+s+\theta^{-\alpha}}\right]^{1-\beta}\right\}$$

a. The sign of the derivative with respect to β turns out to depend on the sign of the expression, $y + c - b$, Given the set up of this model, what do we know about this sign.

b. Someone asked good question, what happens if β=1